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Review Article

A SINGLE SERVER RETRIAL QUEUEING SYSTEM WITH TWO TYPES OF ARRIVALS AND FINITE NUMBER OF RECURRENT REPEATED CUSTOMERS

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ABSTRACT

A retrial queueing system with two types of arrivals and K recurrent calls has been considered. Type I customers arrive in batches of size k with probabilities c_k and type II customers arrive singly according to two Poisson processes with rates $\lambda_1 \bar{c} = \lambda_1 \sum_{k=1}^{\infty} k c_k$ and λ_2 respectively. The retrial orbit consists of transit type II customers and a fixed number K recurrent customers. Service times distributions are identical independent distributions (transit and recurrent) and are different for type I customers and the customers from the orbit, where as the retrial time for both transit and recurrent customers are independently and exponentially distributed with different rates. The customer served by a single server. For this model, the join distribution of the number customers of type I, type II and recurrent in closed form has been obtained. Some particular models and operating characteristics are obtained. A numerical study is also carried out.

INTRODUCTION

In the last three decades there has been a significant contribution in the area of retrial queueing theory. For detailed survey one can see Yang and Templeton (1987) and Falin (1992). Choi and Park (1990) investigated an M/G/1 retrial queue with two types of customers in which the service time distribution for both types of customers are the same. Khailal et al. (1992) investigated the above model at Markovian level in detail. Falin et al.(1993) investigated a similar model, in which they assumed different service time distributions for both types of customers. In 1995, Choi et al. studied an M/G/1 retrial queueing with two types of customers and finite capacity. Choi and Chang (1999) investigated $M_1, M_2/G/1$ retrial queue with recurrent calls in the retrial group, in which they obtained generating function of queue lengths by using supplementary variable technique. Kalyanaraman and Srinivasan (2003), considered an M/G/1 queue with two types of arrivals and recurrent repeated arrivals. For this model they obtained the joint distribution of number of calls in the priority queue and in the retrial group in closed form.

Kalyanaraman and Srinivasan (2004), studied an M/G/1 retrial queue with geometric loss and with type I batch arrivals and type II single arrivals. In 2011 the author with Thillaigovindan analyzed a feedback retrial queueing system with two types of arrivals and the type I arrival being batch arrival of fixed size K. Kalyanaraman (2012 (a)) investigated a feedback retrial queue with type I arrival being batch arrivals and type II arrival single arrivals. The same author (2012 b) analyzed a feedback retrial queueing system with two types of batch arrivals. Ioannis Dimitrious (2013) investigated a batch arrival priority queue with recurrent repeated demands, admission control and hybrid failure recovery discipline. This article deals a retrial queue with two types of customers, in which type I customers arrives in batches of variable size where as type II customers arrive singly. In addition the orbit consists of a fixed number of recurrent customers. In section 2, we describe the system mathematically. In section 3, we obtain the joint probability generating function for the number of customers in the priority queue and in the retrial group when server is busy as well as idle. The expressions for some particular models are deduced in section 4. Some operating characteristics are derived in section 5 and a numerical study is also carried out in section 6.

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The Model

In this article, a single server retrial queueing system with type I batch arrivals, transit type II single arrivals and K recurrent customers has been considered. The type I customers arrives in batches of size k with probability c_k and type II customers arrive singly according to two independent Poisson processes with rates $\lambda_1 \bar{c} = \lambda_1 \sum_{k=1}^{\infty} k c_k$ and λ_2 respectively. When the arriving type I customers are blocked due to server being busy, they are queued in a priority queue of infinite capacity. Otherwise, any one of the customer in the batch being served, the other customers in the batch are queued in the priority queue of infinite capacity. As soon as the server is free, the customer in the priority queue are served using FCFS rule. If type II customers, upon arrival finds the server busy, they enter in to an orbit of infinite capacity in order to seek service again after random amount of time. On the other hand, if the type II customer finds the server free on arrival the customer occupies the server and leaves the system after service completion. In addition, the orbit consists of a fixed number of K recurrent customers. The recurrent customers, which finds the server free, occupies and return to the retrial orbit after service completion. The orbit consist of $(j-K)$ transit customers and K recurrent customers. The retrial time for each transit and recurrent customers are exponentially distributed with rates α_1 and α_2. These retrial rates depend on the number of transit customers and the recurrent customers and is independent of all previous retrial times and all other stochastic processes related to the system.

The customers in the retrial orbit are served, if there are no customers in the priority queue. It is assumed that the type I customers are non-preemptive priority over transit type II customers and recurrent customers. The service times are independent, identically distributed and are different for both priority customers and the customers in the orbit. The density functions are respectively

$b_1(x), b_{21}(x)$ and the Laplace transformation of the distribution function $B_k(x)$ are $B_k^*(s) = \int_0^{\infty} e^{-sx} b_k(x)dx,\ k = 1,2.$

The inter arrival time, service time and retrial times are mutually independent random variables. As an example, in a computer communication system, type I customers are identified as incoming messages and transit type II customers are identified as out going messages, on the other hand, there are fixed number of messages in CPU which are taken as K recurrent customers. The model is analyzed using the supplementary variable technique, residual service time of a customer in service.

The Stochastic process related to this model is $\{(\xi(t), N_p(t), N_r(t), S_k(t)) : t \geq 0\}$ where

$N_p(t)$ = number of customers in the priority queue at time t

$N_r(t)$ = number of customers in the retrial group at time t

$\xi(t)$ = the server state at time t and

$$\xi(t) = \begin{cases} 0, \text{whentheserverisidle} \\ 1, \text{whentheserverisbusywithtypeIcustomer} \\ 2, \text{whentheserverisbusywithtypeIIcustomer} \\ 3, \text{whentheserverisbusywithrecurrentcustomer} \end{cases}$$

$S_k(t)$ = the residual service time of a type k customer in service at time t, is a Markov process with state space $\{0,1,2,3\} \times \{0,1,2,3,\cdots\} \times (0,\infty)$ and the corresponding stationary process is $\{(\xi, N_p, N_r, S_k)\}$.

The related probabilities are defined as $q_j(t) = Pr\{\xi(t) = 0, N_r(t) = j\}$,

$p(k,i,j;x,t)dx = Pr\{\xi(t) = k, N_p(t) = i, N_r(t) = j, S_k(t) \varepsilon (x, x+dx)\}$, k=1, 2, 3.

In steady state, the probabilities are defined as $q_j = \lim_{t \to \infty} q_j(t)$, this leads to

$p(k,i,j;x) = \lim_{t\to\infty} p(k,i,j;x,t)$ and the Laplace transformation of $p(k,i,j;x)$ is $p^*(k,i,j;s) = \int_0^\infty e^{-sx} p(k,i,j;x)dx, i = 1,2,3, j \geq 0.$

It is clear that, $p(k,i,j;0) = \int_0^\infty p(k,i,j;x)dx = Pr\{\xi = k, N_p = i, N_r = j\}$ is the steady state probability that there are i customers in the priority queue, j customers in the retrial group and the server is busy with k th-type customer.

For $|Z_1|,|Z_2| \leq 1,$ the following probability generating functions

$$Q(Z_2) = \sum_{j=0}^{\infty} q_j Z_2^j,$$

$$C(Z_1) = \sum_{j=1}^{\infty} c_j Z_1^j,$$

$$D(Z_2) = \sum_{j=1}^{\infty} d_j Z_2^j,$$

$$P^*(k,i,s,Z_2) = \sum_{j=0}^{\infty} p^*(k,i,j,s) Z_2^j; i = 0,1,2,..., k = 1,2,3.,$$

$$P^*(k,s,Z_1,Z_2) = \sum_{i=0}^{\infty} P^*(k,i,s,Z_2) Z_1^i; k = 1,2,3.,$$

$$P(k,i,0,Z_2) = \sum_{j=0}^{\infty} p^*(k,i,j,0) Z_2^j; i = 0,1,2,..., k = 1,2,3.,$$

$P(k,0,Z_1,Z_2) = \sum_{i=0}^{\infty} P(k,i,0,Z_2) Z_1^i$ are defined for the analysis.

The Analysis

Using the mean drift argument of Falin (1984), it can be show that the system is stable if $\rho_1 + \rho_2 < 1$ where $\rho_1 = -\lambda_1 \bar{c} B_1^{*'}(0),$ $\rho_2 = -\lambda_2 B_2^{*'}(0).$

Usual arguments lead to the following differential-difference equations

For $j \geq 0, x \geq 0, i \geq 0$

$$(\lambda + (j-K)\alpha_1 + K\alpha_2) q_j = p(1,0,j;0) + p(2,0,j;0) + p(3,0,j-1;0) \quad (1)$$

$$-p'(1,0,j;x) = -\lambda p(1,0,j;x) + \lambda_1 b_1(x) q_j + b_1(x) p(1,1,j;0) + \lambda_2 p(1,0,j-1;x) \quad (2)$$

$$-p'(1,i,j;x) = -\lambda p(1,i,j;x) + b_1(x) p(1,i+1,j;0) + \lambda_1 \sum_{k=1}^{i} c_k p(1,i-k,j;x)$$
$$+\lambda_2 p(1,i,j-1;x) \quad (3)$$

$$-p'(2,0,j;x) = -\lambda p(2,0,j;x) + \lambda_2 b_2(x) q_j + \lambda_2 p(2,0,j-1;x)$$

$$+(j-K+1)\alpha_1 b_2(x) q_{j+1} \quad (4)$$

$$-p'(2,i,j;x) = -\lambda p(2,i,j;x) + \lambda_1 \sum_{k=1}^{i} c_k p(2,i-k,j;x) + \lambda_2 p(2,i,j-1;x) \quad (5)$$

$$-p'(3,0,j;x) = -\lambda p(3,0,j;x) + \lambda_2 p(3,0,j-1;x) + K\alpha_2 b_2(x) q_{j+1} \quad (6)$$

$$-p'(3,i,j;x) = -\lambda p(3,i,j;x) + \lambda_1 \sum_{k=1}^{i} c_k p(3,i-k,j;x) + \lambda_2 p(3,i,j-1;x) \quad (7)$$

and the normalization condition is,

$$\sum_{i=0}^{\infty} \sum_{j=K-1}^{\infty} \int_0^{\infty} [p(1,i,j;x) + p(2,i,j;x) + p(3,i,j-1;x)]dx + \sum_{j=K}^{\infty} q_j = 1 \quad (8)$$

where $\lambda = \lambda_1 + \lambda_2, q_j = 0$ and $p(\gamma,i,j;x) = 0, i \geq 0, j = K-1, \gamma = 1,2,3.$

Multiplying equation (1) by Z_2^j and then summing over j, we get

$$[\lambda - K(\alpha_1 - \alpha_2)]Q(Z_2) + \alpha_1 Z_2 Q'(Z_2) = P(1,0;0,Z_2) + P(2,0;0,Z_2) + Z_2 P(3,0;0,Z_2) \quad (9)$$

By taking Laplace transformation on equations (2) to (7) and multiplying by Z_2^j and then summing over j, the following equations can be obtained

$$(s - \lambda + \lambda_2 Z_2) P^*(1,0;s,Z_2) = P(1,0;0,Z_2) - \lambda_1 B_1^*(s) Q(Z_2) - B_1^*(s) P(1,1;0,Z_2) \quad (10)$$

$$(s - \lambda + \lambda_2 Z_2) P^*(1,i;s,Z_2) = P(1,i;0,Z_2) - B_1^*(s) P(1,i+1;0,Z_2)$$

$$-\lambda_1 \sum_{k=1}^{i} c_k P^*(1,i-k;s,Z_2) \quad (11)$$

$$(s - \lambda + \lambda_2 Z_2) P^*(2,0;s,Z_2) = P(2,0;0,Z_2) - \lambda_2 B_2^*(s) Q(Z_2) - \alpha_1 B_2^*(s) Q'(Z_2)$$

$$+\frac{K\alpha_1}{Z_2} B_2^*(s) Q(Z_2) \quad (12)$$

$$(s - \lambda + \lambda_2 Z_2) P^*(2,i;s,Z_2) = -\lambda_1 \sum_{k=1}^{i} c_k P^*(2,i-k;s,Z_2) \quad (13)$$

$$(s - \lambda + \lambda_2 Z_2) P^*(3,0;s,Z_2) = P(3,0;0,Z_2) - \frac{K\alpha_2}{Z_2} B_2^*(s) Q(Z_2) \quad (14)$$

$$(s - \lambda + \lambda_2 Z_2) P^*(3,i;s,Z_2) = -\lambda_1 \sum_{k=1}^{i} c_k P^*(3,i-k;s,Z_2) \quad (15)$$

Multiplying equations (11), (13) and (15) by Z_1^i and adding (10), (12) and (14) and summing over i, leads to

$$(s - \lambda + \lambda_1 C(Z_1) + \lambda_2 Z_2) P^*(1;s,Z_1,Z_2) = P(1;0,Z_1,Z_2) - \lambda_1 B_1^*(s) Q(Z_2)$$

$$-\frac{B_1^*(s)}{Z_1} P(1;0,Z_1,Z_2) + \frac{B_1^*(s)}{Z_1} P(1;0,0,Z_2) \quad (16)$$

$$(s-\lambda+\lambda_1 C(Z_1)+\lambda_2 Z_2)P^*(2;s,Z_1,Z_2)=P(2,0;0,Z_2)-\lambda_2 B_2^*(s)Q(Z_2)$$
$$-\alpha_1 B_2^*(s)Q^{'}(Z_2)+\frac{K\alpha_1}{Z_2}B_2^*(s)Q(Z_2) \quad (17)$$

$$(s-\lambda+\lambda_1 C(Z_1)+\lambda_2 Z_2)P^*(3;s,Z_1,Z_2)=P(3,0;0,Z_2)-\frac{K\alpha_2}{Z_2}B_2^*(s)Q(Z_2) \quad (18)$$

By substituting $s=\lambda-\lambda_1 C(Z_1)-\lambda_2 Z_2$ in (16), (17) and (18), we get

$$P(1,0;0,Z_2)=\lambda_1 z_1 Q(Z_2)-\frac{[Z_1-B_1^*(l)]}{B_1^*(l)}P(1;0,Z_1,Z_2) \quad (19)$$

$$P(2,0;0,Z_2)=B_2^*(l)\left[\left(\lambda_2-\frac{K\alpha_1}{Z_2}\right)Q(Z_2)+\alpha_1 Q^{'}(Z_2)\right] \quad (20)$$

$$Z_2 P(3,0;0,Z_2)=K\alpha_2 B_2^*(l)Q(Z_2) \quad (21)$$

where $l=\lambda-\lambda_1 C(Z_1)-\lambda_2 Z_2$

Using equations (19), (20) and (21) in (9) and on simplification one can get the following equation

$$\left[\lambda-\lambda_1 Z_1-\lambda_2 B_2^*(l)+K\alpha_2\left(1-B_2^*(l)\right)-K\alpha_1\left(1-\frac{B_2^*(l)}{Z_2}\right)\right]Q(Z_2)$$
$$+\alpha_1\left[Z_2-B_2^*(l)\right]Q^{'}(Z_2)=\frac{B_1^*(l)-Z_1}{B_1^*(l)}P(1;0,Z_1,Z_2) \quad (22)$$

Now we define $f(Z_1,Z_2)=\frac{B_1^*(l)-Z_1}{B_1^*(l)}$ for each fixed $Z_2, |Z_2|\le 1,$ by Rouche's theorem, there is a unique solution $Z_1=h(Z_2)$ of the equation $f(Z_1,Z_2)=0,$ now (22) becomes

$$Q^{'}(Z_2)=\left\{\frac{\lambda-\lambda_1 h(Z_2)-\lambda_2 U(Z_2)+K\alpha_2(1-U(Z_2))}{\alpha_1(U(Z_2)-Z_2)}+\frac{K}{Z_2}\right\}Q(Z_2) \quad (23)$$

where $h(Z_2)$ is the root of the equation $Z_1=B_1^*\left(\lambda-\lambda_1 C(Z_1)-\lambda_2 Z_2\right)$ and $U(Z_2)=B_2^*\left(\lambda-\lambda_1 C(Z_1)-\lambda_2 Z_2\right)$

Using equation (23) in (22), it can be seen that

$$P(1;0,Z_1,Z_2)=\frac{\left\{L[Z_2-B_2^*(l)]+R[U(Z_2)-Z_2]\right\}B_1^*(l)}{[B_1^*(l)-Z_1][U(Z_2)-Z_2]}Q(Z_2) \quad (24)$$

where $L=\lambda-\lambda_1 h(Z_2)-\lambda_2 U(Z_2)-K\alpha_1\left(1-\frac{U(Z_2)}{Z_2}\right)+K\alpha_2(1-U(Z_2))$,

$$R=\lambda-\lambda_1 Z_1-\lambda_2 B_2^*(l)-K\alpha_1\left(1-\frac{B_2^*(l)}{Z_2}\right)+K\alpha_2(1-B_2^*(l))$$

Using equation (24) in (19), leads to

$$P(1;0,0,Z_2)=\frac{[\lambda_1 Z_1+R][U(Z_2)-Z_2]+L[Z_2-B_2^*(l)]}{[U(Z_2)-Z_2]}Q(Z_2) \tag{25}$$

Using equation (23) in (20), leads to

$$P(2;0,0,Z_2)=\frac{\left\{\left(\lambda_2-\frac{K\alpha_1}{Z_2}\right)[U(Z_2)-Z_2]+L\right\}B_2^*(l)}{[U(Z_2)-Z_2]}Q(Z_2) \tag{26}$$

The general solution of the differential equation (23) is

$$Q(Z_2)=Q(1)exp\left\{\frac{-1}{\alpha_1}\int_{Z_2}^{1}\frac{T}{U(x)-x}dx\right\} \tag{27}$$

where $T=\lambda-\lambda_1 h(x)-\lambda_2 U(x)-K\alpha_1\left(1-\frac{U(x)}{x}\right)+K\alpha_2(1-U(x))$ and $Q(1)$ is a constant, which is the probability that the server is idle.

Putting $s=0$ in equations (10) and (11) and summing over i, leads to

$$\lambda_2(Z_2-1)\sum_{i=0}^{\infty}P^*(1,i;0,Z_2)=P(1,0;0,Z_2)-\lambda_1 Q(Z_2) \tag{28}$$

Putting $s=0$ in equations (12) and (13) and summing over i, which leads to

$$\lambda_2(Z_2-1)\sum_{i=0}^{\infty}P^*(2,i;0,Z_2)=P(2,0;0,Z_2)-[\lambda_2-\frac{K\alpha_1}{Z_2}]Q(Z_2)-\alpha_1 Q'(Z_2) \tag{29}$$

Putting $s=0$ in equations (14) and (15) and summing over i, which leads to

$$\lambda_2 Z_2(Z_2-1)\sum_{i=0}^{\infty}P^*(3,i;0,Z_2)=Z_2 P(3,0;0,Z_2)-K\alpha_2 Q(Z_2) \tag{30}$$

Adding equations (28)-(30) and using (9), which leads to

$$\lambda_2\sum_{i=0}^{\infty}\left[\sum_{k=1}^{2}P^*(k,i;0,Z_2)+Z_2\sum_{i=0}^{\infty}P^*(3,i;0,Z_2)\right]=\alpha_1 Q'(Z_2)-\frac{K\alpha_1}{Z_2}Q(Z_2)$$

Evaluating at $Z_2=1$ and using the normalization condition, above equation leads to

$$Q'(1)=\frac{1}{\alpha_1}[\lambda_2-Q(1)(\lambda_2-K\alpha_1)] \tag{31}$$

Putting $Z_2=1$ in equation (23), we get

$$Q'(1)=\frac{Q(1)}{\alpha_1\bar{c}(1-\rho_1-\rho_2)}[\lambda_2(\rho_1+\rho_2\bar{c})+K\alpha_1\bar{c}(1-\rho_1-\rho_2)+K\alpha_2\rho_2\bar{c}] \tag{32}$$

From equation (31) and (32), leads to

$$P_I = Q(1) = \frac{\lambda_2 \bar{c}(1-\rho_1-\rho_2)}{[\lambda_2(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} \tag{33}$$

is the probability that the server is idle. In steady state, the probability generating function of number of customers in the orbit when the server is idle is obtained from equations (33) and (27).

Substituting $s = 0$ in equation (16)

$$P^*(1;0,Z_1,Z_2) = \frac{P(1;0,Z_1,Z_2)(1-Z_1)+\lambda_1 Z_1 Q(Z_2) - P(1,0,0;Z_2)}{lZ_1} \tag{34}$$

(34) together with equations (24) and (19) yields the joint probability generating function of the number of customers in the priority queue and in the orbit when the server is busy with type I customer and is

$$P^*(1;0,Z_1,Z_2) = \frac{[1-B_1^*(l)][L(Z_2-B_2^*(l))+R(U(Z_2)-Z_2)]}{l[B_1^*(l)-Z_1][U(Z_2)-Z_2]}Q(Z_2) \tag{35}$$

Putting $s = 0$ in equation (17),

$$P^*(2;0,Z_1,Z_2) = \frac{\left[\lambda_2 - \frac{K\alpha_1}{Z_2}\right]Q(Z_2)+\alpha_1 Q'(Z_2) - P(2;0,0,Z_2)}{l} \tag{36}$$

(36) together with equations (23) and (26) yields the joint probability generating function of the number of customers in the priority queue and in the orbit when the server is busy with type II customer and is

$$P^*(2;0,Z_1,Z_2) = \frac{[1-B_2^*(l)][\lambda-\lambda_1 h(Z_2)-\lambda_2 Z_2+K\alpha_2(1-U(Z_2))]}{l[U(Z_2)-Z_2]}Q(Z_2) \tag{37}$$

Putting $s = 0$ in equation (18),

$$P^*(3;0,Z_1,Z_2) = \frac{1}{l}\left\{\frac{K\alpha_2}{Z_2}Q(Z_2) - P(3;0,0,Z_2)\right\} \tag{38}$$

(38) together with equation (21) yields the joint probability generating function of the number of customers in the priority queue and in the orbit when the server is busy with K recurrent customers and is

$$P^*(3;0,Z_1,Z_2) = \frac{K\alpha_2[1-B_2^*(l)]}{lZ_2}Q(Z_2) \tag{39}$$

Thus we have the following theorem.

Theorem 1: The stationary distribution of $\{(\xi, N_p, N_r, S_k)\}$ has the following generating functions

$$Q(Z_2) = \frac{\lambda_2 \bar{c}(1-\rho_1-\rho_2)}{[\lambda_2(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} exp\left\{\frac{1}{\alpha_1}\int_1^{Z_2}\frac{T}{U(x)-x}dx\right\}$$

$$P^*(1;0,Z_1,Z_2) = \frac{[1-B_1^*(l)][L(Z_2-B_2^*(l))+R(U(Z_2)-Z_2)]}{l[B_1^*(l)-Z_1][U(Z_2)-Z_2]}Q(Z_2)$$

$$P^*(2;0,Z_1,Z_2)=\frac{[1-B_2^*(l)][\lambda-\lambda_1 h(Z_2)-\lambda_2 Z_2+K\alpha_2(1-U(Z_2))]}{l[U(Z_2)-Z_2]}Q(Z_2)$$

$$P^*(3;0,Z_1,Z_2)=\frac{K\alpha_2[1-B_2^*(l)]}{lZ_2}Q(Z_2)$$

Corollary 1

The probability that the server busy is

$$\begin{aligned}P_B &= P^*(1;0,1,1)+P^*(2;0,1,1)+P^*(3;0,1,1)\\ &=\frac{[\lambda_2(\rho_1+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]}{[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}\end{aligned}$$

Particular Models

By taking particular values to some parameters of the above model, the following models can be obtained:

(i) When $K=0, c_i=0, i\neq 1,$ and $B_1(x)=B_2(x)=B(x)$ the system coincides with that of Choi and Park (1990).

(ii) When $K=0, c_i=0, i\neq 1,$ the above results coincides with the results of Falin et al. (1993).

(iii) When $d_i=0=c_i, i\neq 1,$ and $B_1(x)=B_2(x)=B(x)$ the system coincides with that of Kalyanaraman and Srinivasan (2003).

(iv) When $q_i=0, i=1,2,$ the system coincides with Kalyanaraman (2012).

Operating characteristics

Using straight forward calculations, the operating characteristics like the mean number of customers in the priority queue and the mean number of customers in the orbit, the mean waiting time of a tagged type I customer in the priority queue and the mean waiting time of a tagged type II customer in the orbit have been calculated. Putting $Z_2=1$ in equations (35), (37) and (39), after the differential coefficient with respect to Z_1 has been obtained and then taking $Z_1=1$.

$$\begin{aligned}\lim_{Z_1\to 1} P^{*'}(1;0,Z_1,1) &= \frac{1}{2\bar{c}(1-\rho_1)[\lambda_2(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}\{\rho_1 c_2[\lambda_2(\rho_1+\rho_2\bar{c})\\ &\quad +K\alpha_2\rho_2\bar{c}]+\lambda_1^2\bar{c}^3\beta_1[\lambda_2(1-\rho_2+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]+\lambda_1\lambda_2\rho_1\beta_2\bar{c}^2\\ &\quad \times[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\bar{c}(1-\rho_1)]\}\end{aligned} \tag{40}$$

$$\lim_{Z_1\to 1} P^{*'}(2;0,Z_1,1)=\frac{\lambda_1\lambda_2\bar{c}\beta_2}{2} \tag{41}$$

$$\lim_{Z_1\to 1} P^{*'}(3;0,Z_1,1)=\frac{\lambda_1\lambda_2\bar{c}^2K\alpha_2\beta_2(1-\rho_1-\rho_2)}{2[\lambda_2(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} \tag{42}$$

Putting $Z_1=1$ in equations (35), (37) and (39), after the differential coefficient with respect to Z_2 has been obtained and then taking $Z_2=1$.

$$\lim_{Z_2\to 1} P^{*'}(1;0,1,Z_2) = \frac{\lambda_2(D_1+(1-\rho_2)D_2)}{2\lambda_1\bar{c}^2(1-\rho_1)(1-\rho_1-\rho_2)[\lambda_2(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} + \frac{\rho_1[\lambda_2(1-\rho_2+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]}{\alpha_1\bar{c}(1-\rho_1-\rho_2)[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} \times[\lambda_2(\rho_1+\rho_2\bar{c})+K\alpha_1\bar{c}(1-\rho_1-\rho_2)+K\alpha_2\rho_2\bar{c}] \quad (43)$$

where $D_1=\lambda_1\lambda_2\bar{c}^2\beta_2\rho_1(2-\rho_1-\rho_2)[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\bar{c}(1-\rho_1)]$
$D_2=\lambda_1^2\bar{c}^3\beta_1[\lambda_2(1-\rho_2+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]+\rho_1^2c_2[\lambda_2(\rho_1+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]$

$$\lim_{Z_2\to 1} P^{*'}(2;0,1,Z_2) = \frac{\lambda_2\rho_2\{D_2+\lambda_1\lambda_2\beta_2\bar{c}^2[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\bar{c}(1-\rho_1)]\}}{2\lambda_1\bar{c}^2(1-\rho_1)(1-\rho_1-\rho_2)[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} + \frac{\lambda_2^2\beta_2}{2} + \frac{\rho_2[\lambda_2(\rho_1+\rho_2\bar{c})+K\alpha_1\bar{c}(1-\rho_1-\rho_2)+K\alpha_2\rho_2\bar{c}]}{\alpha_1\bar{c}(1-\rho_1-\rho_2)} \quad (44)$$

$$\lim_{Z_2\to 1} P^{*'}(3;0,1,Z_2) = \frac{K\alpha_2}{2\alpha_1[\lambda_2(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}\{\alpha_1\bar{c}(1-\rho_1-\rho_2)[\lambda_2^2\beta_2+2\rho_2 \times(K-1)]+2\rho_2[\lambda_2(\rho_1+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]\} \quad (45)$$

From (32) and (33)

$$Q'(1) = \frac{\lambda_2[\lambda_2(\rho_1+\rho_2\bar{c})+K\alpha_1\bar{c}(1-\rho_1-\rho_2)+K\alpha_2\rho_2\bar{c}]}{\alpha_1[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} \quad (46)$$

Theorem: 2: The marginal means of the two dimensional random variables (N_p, N_r) has been obtained as:
Mean number of customers in the priority queue is

$$E(N_p) = \frac{1}{2\bar{c}(1-\rho_1)[\lambda_2(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}\{\rho_1c_2[\lambda_2(\rho_1+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}] + \lambda_1^2\bar{c}^3\beta_1[\lambda_2(1-\rho_2+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]+\lambda_1\lambda_2\beta_2\bar{c}^2[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c}) + K\alpha_2\bar{c}(1-\rho_1)]\} \quad (47)$$

Mean number of customers in the orbit is

$$E(N_r) = \frac{\{\lambda_1\lambda_2^2\bar{c}^2\beta_2[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\bar{c}(1-\rho_1)]+\lambda_2D_2\}}{2\lambda_1\bar{c}^2(1-\rho_1)(1-\rho_1-\rho_2)[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} + K + \frac{[\lambda_2(\rho_1+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]}{\alpha_1\bar{c}(1-\rho_1-\rho_2)} - \frac{K\alpha_2\rho_2\bar{c}(1-\rho_1-\rho_2)}{[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} \quad (48)$$

Mean busy period is

$$E(T_b) = \frac{[\lambda_2(\rho_1+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]}{\lambda\lambda_2\bar{c}(1-\rho_1-\rho_2)} \quad (49)$$

Mean waiting time of a tagged type I customer in the priority queue is

$$E(W_p) = \frac{1}{2\lambda_1\bar{c}^2(1-\rho_1)[\lambda_2(\bar{c}+\rho_1-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}\{\rho_1 c_2[\lambda_2(\rho_1+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}] + \lambda_1^2\bar{c}^3\beta_1[\lambda_2(1-\rho_2+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}]+\lambda_1\lambda_2\beta_2\bar{c}^2[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c}) + K\alpha_2\bar{c}(1-\rho_1)]\} \quad (50)$$

Mean waiting time of a tagged type II customer in the orbit is

$$E(W_r) = \frac{\{\lambda_1\lambda_2\bar{c}^2\beta_2[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\bar{c}(1-\rho_1)]+D_2\}}{2\lambda_1\bar{c}^2(1-\rho_1)(1-\rho_1-\rho_2)[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]}+\frac{K}{\lambda_2} + \frac{\{\lambda_2(\rho_1+\rho_2\bar{c})+K\alpha_2\rho_2\bar{c}\}}{\lambda_2\alpha_1\bar{c}(1-\rho_1-\rho_2)} - \frac{K\alpha_2\rho_2\bar{c}(1-\rho_1-\rho_2)}{\lambda_2[\lambda_2(\rho_1+\bar{c}-\rho_1\bar{c})+K\alpha_2\rho_2\bar{c}]} \quad (51)$$

Proof

Equations (47) is obtained by adding (40), (41) and (42), equations (48) is obtained by adding (43), (44), (45) and (46). Busy period T_b is the length of the time interval that keeps the server busy continuously and this continues till the instant the server becomes free again and let T_0 be the length of the idle period. For this model, T_b and T_0 generates an alternating renewal process and therefore

$$\frac{E(T_b)}{E(T_0)} = \frac{Pr\{T_b\}}{1-Pr\{T_b\}} = \frac{P_B}{1-P_B}$$

But $E(T_0) = \frac{1}{\lambda}$

$$E(T_b) = \frac{P_B}{\lambda(1-P_B)} \quad (52)$$

Using corollary on equation (52), we get equation (49). Equation (50) is obtained using $\frac{E(N_p)}{\lambda_1\bar{c}}$ and Equation (51) obtained by using $\frac{E(N_r)}{\lambda_2}$.

Numerical study

In this section, some numerical examples related to the model analyzed in this article are given. For the sake of convenience, it has been assumed that the type I and type II service times are exponentially distributed random variables with mean $\frac{1}{\mu_1}$ and $\frac{1}{\mu_2}$ and the batch size distributions are geometrically distributed with parameters θ_1. In order to see the effect of the parameters type I service rate μ_1, type II service rate μ_2, type I arrival rate λ_1, type II arrival rate λ_2 on the mean number of customers in the priority queue, the mean number of customers in the orbit, the mean waiting time of a tagged type I customer in the priority queue, the mean waiting time of a tagged type II customer in the orbit, the mean busy period, the probability that the server is idle and the probability that the server is busy of the model discussed in this paper by fixing the values of the retrial rates α_1, α_2 and K, θ_1, θ_2. Some numerical results are obtained. The results are presented in graphs and tables. The figures 1 to 3 (4 to 6) represents the surface of type I arrival rate, type I service rate (type II service rate) and the mean number of customers in the priority queue, and the mean number of customers in the orbit and the mean busy period respectively. The figures 7 to 9 (10 to 12) represents the surface of type II arrival rate, type I service rate (type II service rate)and the mean number of customers in the priority queue, the mean number of customers in the orbit, and the mean busy period respectively. The figures 13 and 14 (19 and 20) represent the surface of type I arrival rate, type I service rate (type II arrival rate, type II service rate) and the mean waiting time of a tagged type I customer in the priority queue, mean waiting time of tagged type II customer in the orbit respectively (mean waiting time of a tagged type I customer in the priority queue, mean waiting time of type II customer in the orbit respectively). Figures 1, 4 shows that as type I arrival rate increases, the mean number of customers in the priority queue increases steadily for small value of type I service rate, that is, 4 whereas the increment is too small for comparatively big values, that is, like 8. The same situation has been encounter in the case of type II service rate. Figure 7 represents the surface of type II arrival rate, type I service rate and the mean number of customers in the priority queue is convex surface for increasing the values of type I service rate, type II service rate and the mean number of customers in the priority queue is convex with respect to type II service rate but increases with respect to type II arrival rate. The surface of the mean number of customers in the priority queue, type I arrival rate (type II arrival rate) and type I service rate in the figures 2 (8) is almost a convex surface at the comparatively large value of type I arrival rate and the small value of type I service rate (is a flat and increasing surface for increasing value of type II arrival rate). Figures 5 and 11 represents the surface area of the mean number of customers in the orbit versus type I arrival rate versus type II service rate and the mean number of customers in the orbit versus type II arrival rate versus type II service rate respectively. The first surface is convex with respect to type II service rate but slightly increases for the increasing values of type I arrival rate whereas the second surface is again convex with respect to type II service rate and increases with respect to type II arrival rate for smallest values of type II service rate. Figures 3, 6, 9 and 12 represents the surface of the mean busy period versus type I arrival rate or type II arrival rate versus type I service rate or type II service rate. All the surfaces are convex with respect to service rate and are increases for increasing values of arrival rate.

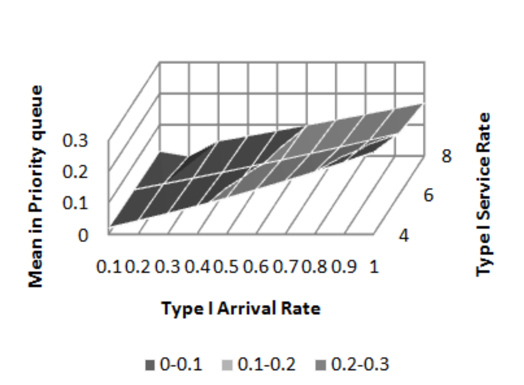


Figure.1. Mean number of customers in priority Queue for Different Type I arrival, Type I service rate

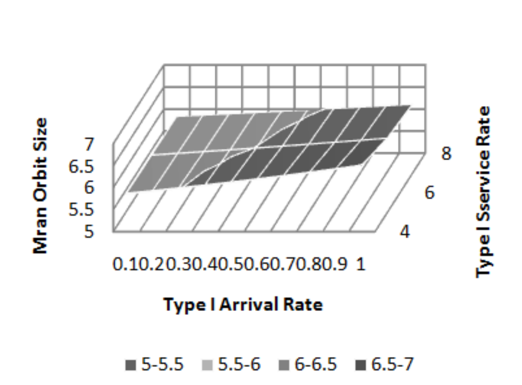


Figure.2. Mean number of customers in orbit for Different Type I arrival , Type I service rate

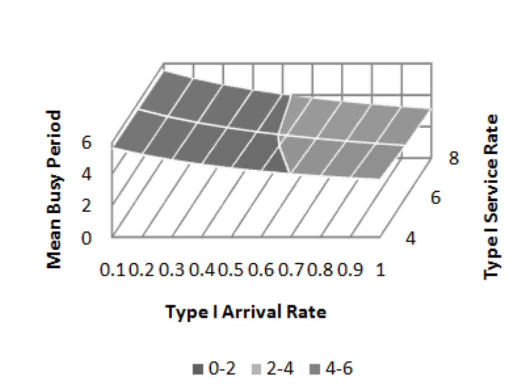


Figure.3. Mean Busy period for Different Type I arrival, Type I service rate

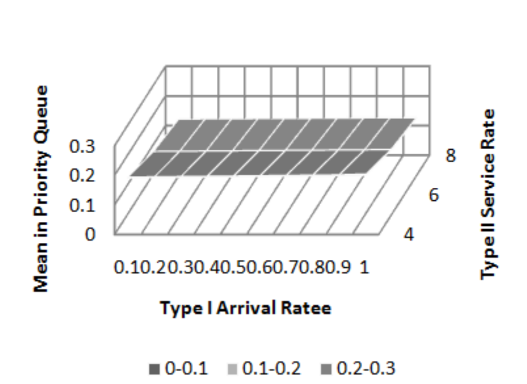


Figure.4. Mean number of customers in priority queue for Different Type I arrival, Type II service rate

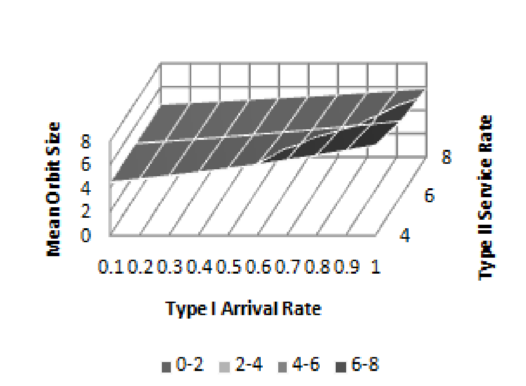


Figure.5. Mean number of customers in the orbit for Different Type I arrival, Type II service rate

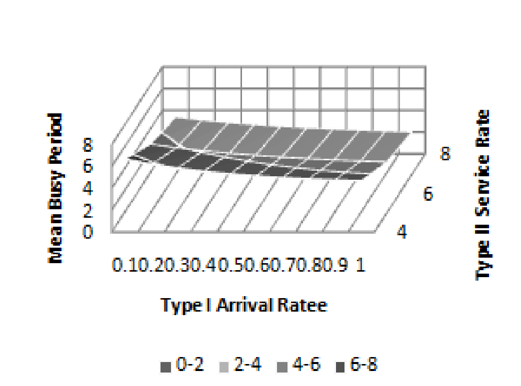


Figure.6. Mean busy periodfor Different Type I arrival, Type II service rate

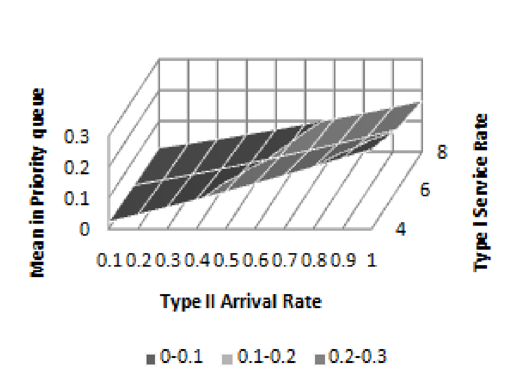


Figure.7. Mean number of customers in priority Queue for Different Type II arrival, Type I service rate

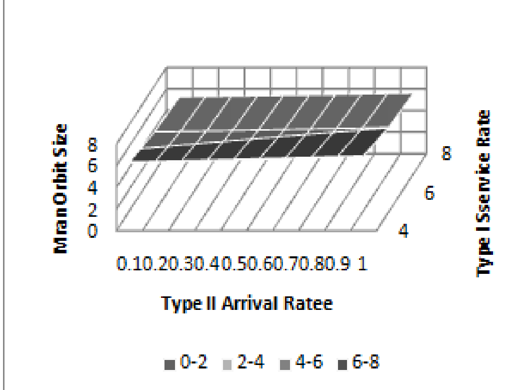


Figure.8. Mean number of customers in orbit for Different Type II arrival ,Type I service rate

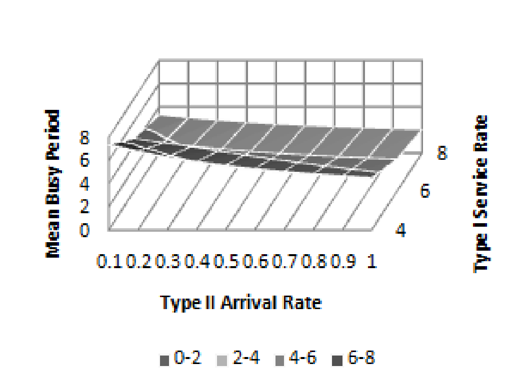


Figure.9. Mean Busy period for Different Type I arrival, Type I service rate

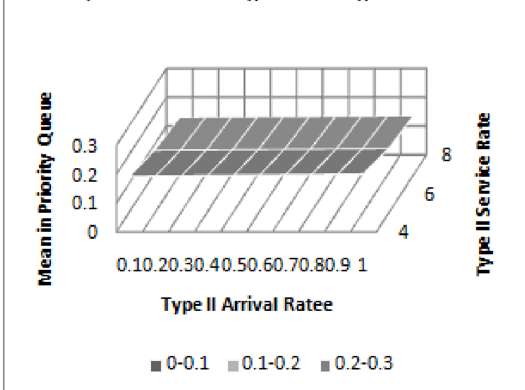


Figure.10. Mean number of customers in priority queue for Different Type IIarrival ,Type II service rate

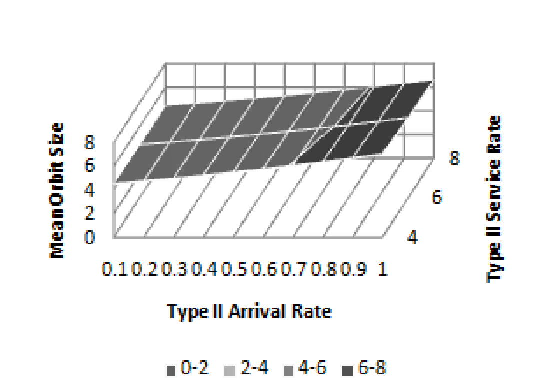


Figure.11. Mean number of customers in the orbit for Different Type I arrival ,Type II service rate

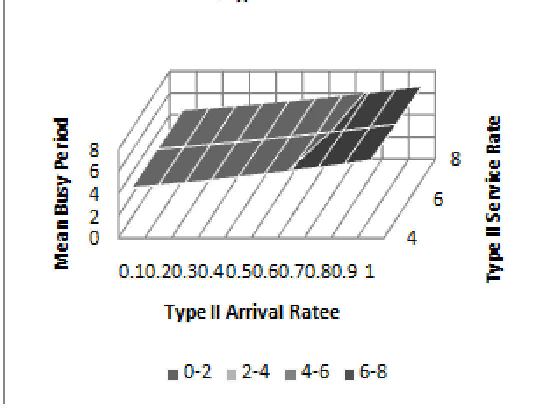


Figure.12. Mean busy periodfor Different Type II arrival, Type II service rate

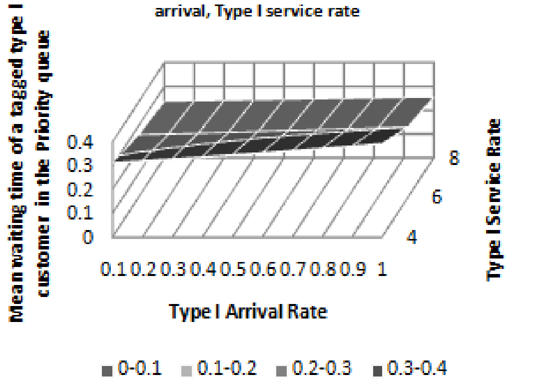


Figure.13. Mean waiting time of a tagged type I customer in the priority queue for Different Type I arrival, Type I service rate

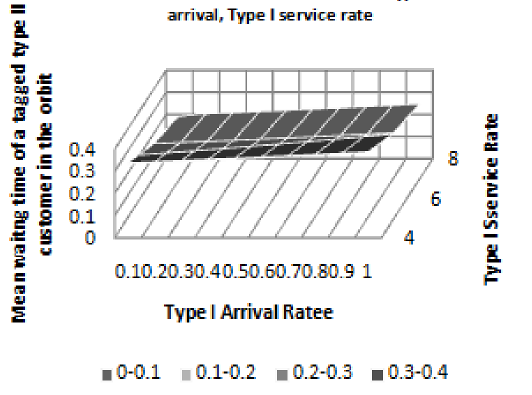


Figure.14. Mean waiting time of a tagged type II customer in the orbit for Different Type I arrival, Type I service rate

Figure.15. Mean waiting time of a tagged type I customer in the priority queue for Different Type I arrival, Type II service rate

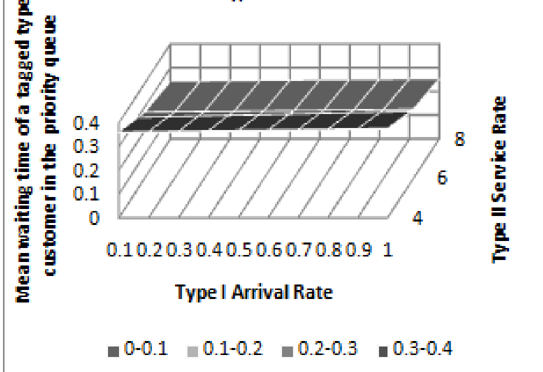


Figure.16. Mean waiting time of a tagged type II customers in the orbit for Different Type I arrival, Type II service rate

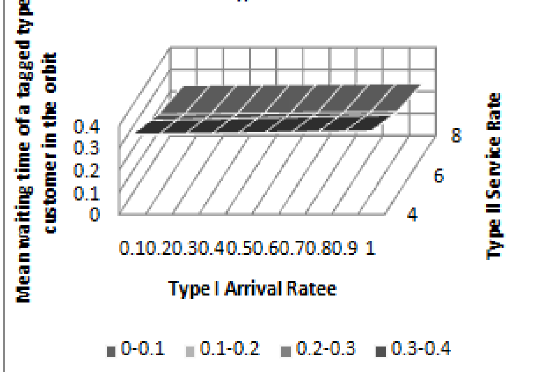


Figure.17. Mean waiting time of a tagged type I customers in the priority queue for Different Type II arrival ,Type I service rate

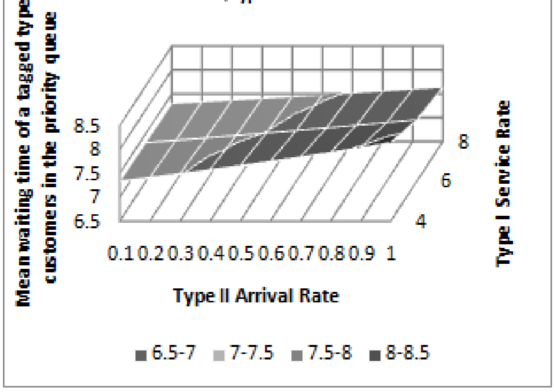


Figure.18. Mean waiting time of a tagged type II customers in the orbit for Different Type II arrival, Type I service rate

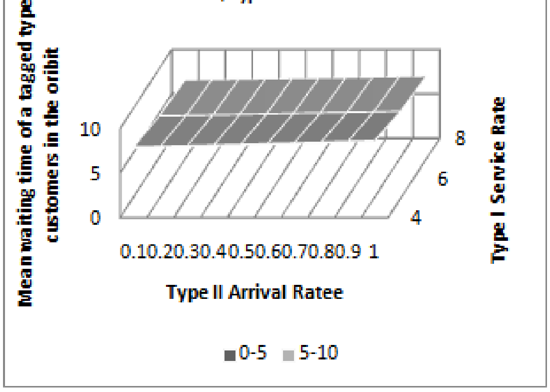


Figure.19. Mean waiting time of a tagged type I customer in the priority queue for Different Type II arrival, Type II service rate

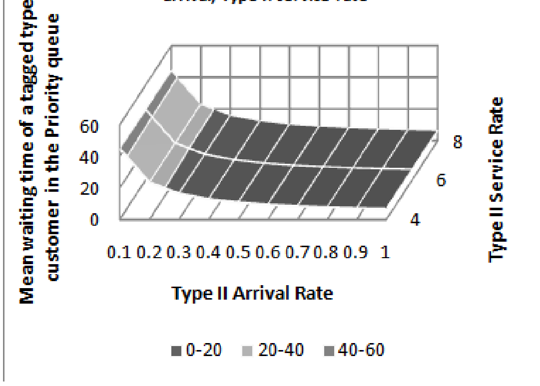
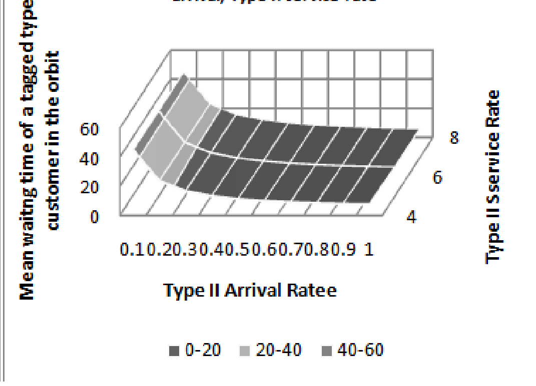


Figure.20. Mean waiting time of a tagged type II customer in the orbit for Different Type II arrival, Type II service rate



Figures 13 and 14 shows that as the type I arrival rate increases the mean waiting time of a tagged type I customer in the priority queue and the mean waiting time of a tagged type II customer in the orbit increases whereas with respect type I service rate, the figures are concave for large value of type I arrival rate. Figures 15 and 16 shows with respect to type I arrival rate, the mean waiting time of a tagged type I customer in the priority queue, the mean waiting time of a tagged type II customer in the orbit are increasing function but with respect to type II service rate their concave in nature. Figurers 17 and 18 shows the figures the mean waiting time of a tagged type I customer in the priority queue, the mean waiting time of a tagged type II customer in the orbit are increasing with respect to type II arrival rate but concave with respect to type I service rate. Finally figures 19 and 20 respects the mean waiting time of a tagged type I customer in the priority queue, the mean waiting time of a tagged type II customer in the orbit are increases with respect to type II arrival rate and concave with respect to type I service rate. Tables 1-4 shows the probabilities that the server is idle and the server is busy for the various values of $\lambda_1, \lambda_2, \mu_1$ and μ_2. From tables it can be seen that, for increasing values of type I arrival rate and type II arrival rate, for fixed values of type I service rate and type II service rate the idle probabilities decreases whereas the busy probabilities increases as expected.

Table 1. The probabilities P_I & P_B

$\alpha_1 = 3.0, \alpha_2 = 4.0,\ \lambda_2 = 0.8,\ \mu_2 = 5.0,\ \theta_1 = 0.4$						
λ_1	$\mu_1 = 4.0$		$\mu_1 = 6.0$		$\mu_1 = 8.0$	
	P_I	P_B	P_I	P_B	P_I	P_B
0.1	0.1644	0.8356	0.1656	0.8344	0.1662	0.8338
0.2	0.1608	0.8392	0.1632	0.8368	0.1644	0.8356
0.3	0.1572	0.8428	0.1608	0.8392	0.1626	0.8374
0.4	0.1536	0.8464	0.1584	0.8416	0.1608	0.8392
0.5	0.1501	0.8499	0.1560	0.8440	0.1590	0.8410
0.6	0.1465	0.8535	0.1536	0.8464	0.1572	0.8428
0.7	0.1430	0.8570	0.1513	0.8487	0.1554	0.8446
0.8	0.1395	0.8605	0.1489	0.8511	0.1536	0.8464
0.9	0.1360	0.8640	0.1465	0.8535	0.1519	0.8481
1.0	0.1325	0.8675	0.1442	0.8558	0.1501	0.8449

Table 2. The probabilities P_I & P_B

$\alpha_1 = 3.0, \alpha_2 = 4.0,\ \lambda_2 = 0.8,\ \mu_1 = 5.0,\ \theta_1 = 0.4$						
λ_1	$\mu_2 = 4.0$		$\mu_2 = 6.0$		$\mu_2 = 8.0$	
	P_I	P_B	P_I	P_B	P_I	P_B
0.1	0.1310	0.8690	0.1966	0.8034	0.2529	0.7471
0.2	0.1286	0.8714	0.1933	0.8067	0.2486	0.7514
0.3	0.1262	0.8738	0.1899	0.8101	0.2443	0.7557
0.4	0.1239	0.8761	0.1865	0.8135	0.2401	0.7599
0.5	0.1215	0.8785	0.1832	0.8168	0.2358	0.7642
0.6	0.1192	0.8808	0.1799	0.8201	0.2316	0.7684
0.7	0.1169	0.8831	0.1766	0.8234	0.2274	0.7726
0.8	0.1145	0.8855	0.1733	0.8267	0.2233	0.7767
0.9	0.1122	0.8878	0.1700	0.8300	0.2191	0.7809
1.0	0.1099	0.8901	0.1667	0.8333	0.2150	0.7850

Table 3. The probabilities P_I & P_B

$\alpha_1 = 3.0, \alpha_2 = 4.0,\ \lambda_1 = 0.8,\ \mu_2 = 5.0,\ \theta_1 = 0.4$						
λ_2	$\mu_1 = 4.0$		$\mu_1 = 6.0$		$\mu_1 = 8.0$	
	P_I	P_B	P_I	P_B	P_I	P_B
0.1	0.1671	0.8329	0.1767	0.8233	0.1815	0.8185
0.2	0.1632	0.8368	0.1727	0.8273	0.1775	0.8225
0.3	0.1592	0.8408	0.1687	0.8313	0.1735	0.8295
0.4	0.1553	0.8447	0.1648	0.8352	0.1695	0.8305
0.5	0.1513	0.8487	0.1608	0.8392	0.1656	0.8344
0.6	0.1474	0.8526	0.1568	0.8432	0.1616	0.8384
0.7	0.1434	0.8566	0.1529	0.8471	0.1576	0.8424
0.8	0.1395	0.8605	0.1489	0.8511	0.1536	0.8464
0.9	0.1355	0.8645	0.1449	0.8551	0.1497	0.8503
1.0	0.1316	0.8684	0.1410	0.8590	0.1457	0.8543

Table 4. The probabilities P_I & P_B

$\alpha_1 = 3.0, \alpha_2 = 4.0,\ \lambda_1 = 0.8,\ \mu_1 = 5.0,\ \theta_1 = 0.4$						
λ_2	$\mu_2 = 4.0$		$\mu_2 = 6.0$		$\mu_2 = 8.0$	
	P_I	P_B	P_I	P_B	P_I	P_B
0.1	0.1434	0.8566	0.1998	0.8002	0.2479	0.7521
0.2	0.1393	0.8607	0.1960	0.8040	0.2444	0.7556
0.3	0.1352	0.8648	0.1922	0.8078	0.2409	0.7591
0.4	0.1311	0.8689	0.1884	0.8116	0.2373	0.7627
0.5	0.1269	0.8731	0.1847	0.8153	0.2338	0.7662
0.6	0.1228	0.8772	0.1809	0.8191	0.2303	0.7697
0.7	0.1187	0.8813	0.1771	0.8229	0.2268	0.7732
0.8	0.1145	0.8855	0.1733	0.8267	0.2233	0.7767
0.9	0.1104	0.8896	0.1695	0.8305	0.2197	0.7803
1.0	0.1063	0.8937	0.1657	0.8343	0.2162	0.7838

Conclusion

In the fore going analysis, an $M/G/1$ queue with retrial customers, with two types of arrivals and with finite number recurrent repeated customers is considered. We obtain the queue length distribution and mean queue length. Extensive numerical work has been carried out to observe the trends of the operating characters of the system.

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