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Research Article

"STUDY AND ANALYSIS OF SANDWICH BEAM WITH GRADED MATERIAL FOR DYNAMIC STABILITY AND FREE VIBRATION"

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ABSTRACT

The research focus on sandwich beam which is graded material as containing in high thermal environment. The aim of this work to find out the stability of dynamic and free vibration for sandwich beam. Here main advantage of the sandwich beam as top layer of functionaly graded material and stainless steel on the bottom.Functionally graded material (FGM) belongs to advanced type material that have unique and different properties from any other material.This sandwich beam is subjected to axial dynamic loaded condition and the layer is having high temprature but other layer of the beam is normal (room temprature). The variation of the temprature in the sandwich beam are linear and non linear. The model of beam is assiocated with FEM and beam element has four degree of freedom and rotation in case of degree of freedom are longitudnal displacement. The thickness of beam are responsible for there displacement and boundries for stable and unstable region are considered in this work. In this study the frequency of the beam also decreases with increase in the thickness parameter, power loe index and temprature. The instability of the beam increases with increase in core thickness parameter, power low index and temprature of the top layer.

INTRODUCTION

System subjected to dyanamic loading damping property has high impoetance. The noise and vibration in beam structure and system is reduced by passive damping meyhod. In aerospace and structure construction, vibration and niose are occure more frequently. The vibration and noise are reduced by viscoelastic material that are sandwich structure between the surface and thin facing of metallic material. Core material is a thin section that do not buckle under the applied load. To avoid buckling under load high modulus of elastic material is required. The top and bottom layer of sandwich beam are called face material. Generally it is a thin layers in sheet form. Tensile and compressive stress are carried out by face material. Functionaly graded material are capable to withstand high temprature. Functionaly graded materials are made of generally ceramic and metallic material. The requirement of metallic material provide structural support and ceramic is provide heat shielding in high temprature condition. A sandwich beam are designed to capable structural load throughout. For this desgn purpose the face sheetn should have sufficient stiffnes to withstand high compressive load and shear stress.the core sheet of this beam should have sufficient stiffness towithstand shear stress due to applied load. Michael Faraday (Faraday, 1831) in 1831 was the first scientist to observe parametric resonance. He observed that when a fluid filled glass is vibrating vertically, the fluid surface is oscillating at half the frequency at which the container is oscillating. Beliaev (Beliaev, 1924) was the first scientist who made a theoretical analysis on the parametric stability of prismatic beams. Di Taranto (Di Taranto, 1965) gave a theory for finding natural frequencies and loss factors for finite length sandwich beams. Brown et al. (1968) studied dynamic stability of uniform bar using finite element method. Mead and Markus (1969) analyzed the forced vibration of a three layered sandwich beam with visco-elastic core by using the method of Di Taranto (Di Taranto, 1965).Asnani and Nakra (1970) has worked on multi-layer simply supported sandwich beams and found out the loss factors and displacement response for beams of different numbers of layers.

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Asani and Nakra (1976) worked on forced vibration of a sandwich beam with visco-elastic core for fixed-fixed and cantilever type end conditions. Nakra (Rao, 1974), (Simsek, 2010) has presented an article on vibration control in visco-elastic material.Asnani and Nakra (1976) analyzed the effect of number of layers and thickness ratio of layers on loss factor of a multi-layer simply supported beam. Ibrahim and his co-workers (Ibrahim and and Barr, 1978)-(Ibrahim, 1985) have presented a review article on linear and non-linear parametric vibration of deterministic and stochastic type. Johnson and his co-workers (Johnson et al., 1981)-(Johnson and Kienholz, 1982) calculated frequencies and loss factor for beams and plates with constrained viscoelastic layer using finite element technique. Ha (1992) developed an exact analysis method for bending and buckling analysis of sandwich beam. Briseghella *et al.* (1998) worked on dynamic stability problems of beams and frames using finite element method. Fasana and Marchesiello (2001) used Rayleigh-Ritz method and found out the frequencies, loss factors and mode shapes for sandwich beams. They used polynomials which satisfied the boundary conditions. Banerjee (2003) used dynamic stiffness matrix and found out the mode shapes and natural frequencies of a three layer sandwich beam. Akhtar and Kadoli, (2008) Stress analysis of SUS 304- ceramics functionally graded beams using third order shear deformation theory. Kapuria *et.all* (2008) Bending and free vibration response of layered functionally graded beams: A theoretical model and its experimental validation. Composite Structures, 82, 392-402. Simsek M.(Kapuria et al., 2008) Vibration analysis of a functionally graded beam under a moving mass by using different beam theories. Composite Structures, 92(4), 904-917. Aydogdu and Taskin, (2013), free vibration analysis of functionally graded beams with simply supported edges. Materials & Design, 28(5), 1651-1656, Bhangale and Ganeshan, (2015) thermo-elastic buckling and vibration behaviour of a functionally graded sandwich beam with constrained viscoelastic core.

MATERIALS AND METHODS

Our analysis is based on two case of temperature distribution on sandwich beam graded material. T_t & T_b are temperature at top and bottom face of sandwich beam respectively. The temperature variation inside sandwich beam due to linear temperature distribution in thickness z- direction is expressed as-

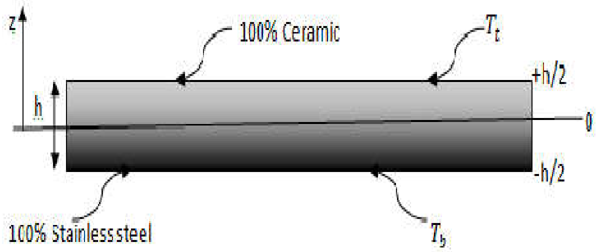


Fig. 1. FGM beam in thermal environment

$$T(z) = T_b + \Delta T\left(\frac{z}{h} + \frac{1}{2}\right) \quad (1)$$

Where $\Delta t = T_t - T_b$ is the temperature gradient between top and bottom layer of sandwich beam material and equals the initial temperature 300K.

The non-linear temperature variation along thickness direction is formulated by one-dimensional heat conduction equation. That is expressed as-

$$\frac{d}{dz}\left(\hat{k}(z)\frac{dT}{dz}\right) = 0 \quad (2)$$

Where $\hat{k}(z)$ denote thermal conductivity of FGM along thickness direction, $T = T_t$ at the top face of beam(z =+h/2) and $T = T_b$ at bottom face of FGM beam (z = -h/2) and initial thickness of FGM beam is (h).

The analytical solution of equation is (2)

$$T(z) = T_t \quad (T_t \quad T_b)\frac{\int_{-h/2}^{z}\frac{1}{\hat{k}_{(z)}}dz}{\int_{-h/2}^{h/2}\frac{1}{\hat{k}_{(z)}}dz} \quad (3)$$

In case of sandwich beam material, the equation (2) is expressed in term of polynomial series-

$$T(\xi) = T_b + \frac{T_t - T_b}{C_{tb}}\left[\left(\frac{z}{2}+\frac{1}{2}\right) \quad \frac{\hat{k}_{tb}}{(n+1)\widehat{k_b}}\left(\frac{z}{2}+\frac{1}{2}\right)^{n+1} + \frac{\hat{k}_{tb}^2}{(2n+1)\widehat{k_b^{23}}}\left(\frac{z}{2}+\frac{1}{2}\right)^{2n+1} \quad \frac{\hat{k}_{tb}^3}{(3n+1)\widehat{k_b^3}}\left(\frac{z}{2}+\frac{1}{2}\right)^{3n+1} + \frac{\hat{k}_{tb}^4}{(4n+1)\widehat{k_b^4}}\left(\frac{z}{2}+\frac{1}{2}\right)^{4n+1} \\ \frac{\hat{k}_{tb}^5}{(5n+1)\hat{k}_b^5}\left(\frac{z}{2}+\frac{1}{2}\right)^{5n+1}\right] \quad (4)$$

Where $\hat{k}_{tb}$ Thermal conductivity of FGM along its thickness, n= power low index of FGM and T_t & T_b are temperature at top and bottom face of sandwich beam respectively. Temperature T_b is taken as 300K.

With

$$C_{tb} = \Bigg[1 \quad \frac{\hat{k}_{tb}}{(n+1)\widehat{k_b}}\left(\frac{z}{2}+\frac{1}{2}\right)^{n+1} + \frac{\hat{k}_{tb}^{2}}{(2n+1)\widehat{k_b^{23}}}\left(\frac{z}{2}+\frac{1}{2}\right)^{2n+1} \quad \frac{\hat{k}_{tb}^{3}}{(3n+1)\widehat{k_b^{3}}}\left(\frac{z}{2}+\frac{1}{2}\right)^{3n+1} + \frac{\hat{k}_{tb}^{4}}{(4n+1)\widehat{k_b^{4}}}\left(\frac{z}{2}+\frac{1}{2}\right)^{4n+1} \quad \frac{\hat{k}_{tb}^{5}}{(5n+1)\hat{k}_b^{5}}\left(\frac{z}{2}+\frac{1}{2}\right)^{5n+1}\Bigg]$$

Where$\hat{k}_{tb} = \hat{k}_t \quad \hat{k}_b$,$\hat{k}_t and \hat{k}_b$ denotes thermal conductivity of the top bottom face of beam respectively and n denotes the volume fraction index of power low.

Generally, the properties of any sandwich material beam changes continuously due to gradual variation of the volume fraction of constituent materials, normally in thickness direction only. Power law function is usually used to show this variation of ceramic material volume fraction. This can be expressed as-

$$g(z) = \left\{\frac{z}{h}+\frac{1}{2}\right\}^{n} \tag{5}$$

Where *n* $(0 \le n \le \infty)$, which shows the profile of material variation along the thickness direction of FGM beam.

The material properties- Young's modulus E, Poisson's ratio υ and Thermal co-efficient of expansion α are the non-linear functions of temperature P (T). Their dependence on temperature can be expressed by the following equation-

$$P(T) = P_o^T\left(P_{-1}^T\frac{1}{T} + 1 + P_1^T T + P_2^T T^2 + P_3^T T^3\right) \tag{6}$$

Where T $=T_o+ \Delta T$, which denotes the environment's temperature, and T_o is the temperature of free stress state i.e T_o=300K; $P_o^T, P_{-1}^T, P_{-1}^T$ *and* P_3^T are the constraints in the cubic equation stating the dependence of temperature by material property.

According to the mixture rule, the effective material properties can be expressed in following equation-

$$P_{eff}(z,T) = P_b(T) + [P_t(T) \quad P_b(T)]g(z) \tag{7}$$

Where $P_{eff}(\xi,T)$ denotes the effective material properties of FGM at temperature T, $P_t(T)$ and $P_b(T)$ denotes the properties of the top and bottom face at temperature T respectively.

In this work, the effective Poisson's ratio υ, Young's modulus E and thermal co-efficient of expansion α are assumed to be dependent on temperature, but the mass density ρ and thermal

Conductivity $\hat{k}$ are not dependent on temperature, i.e.

$$E(z,T) = E_b(T) + [E_t(T) \quad E_b(T)]g(z)$$
$$v(z,T) = v_b(T) + [v(T) \quad v_b(T)]g(z)$$
$$\alpha(z,T) = \alpha_b(T) + [\alpha_t(T) \quad \alpha_b(T)]g(z)$$
$$\rho(z) = \rho_b + [\rho_t \quad \rho_b]g(z)$$
$$\hat{k}(z) = \hat{k}_b + [\hat{k}_t \quad \hat{k}_b]g(z)$$

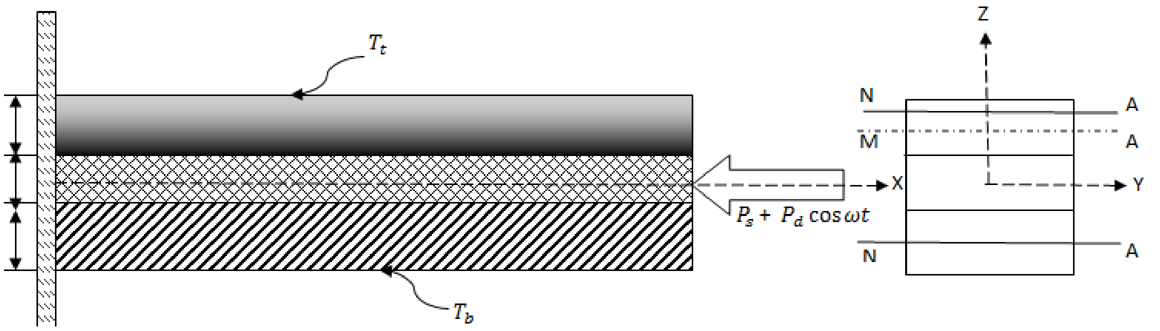


Fig. 2. Three layered sandwich beam consisting top layer as FGM layer, middle layer as viscoelastic layer and bottom layer as pure metal, subjected to dynamic loading

Fig.2 shows a three layered symmetric sandwich beam of length L, consisting of top layer as constraining layer made up of Functionally Graded Material, middle layer made up of viscoelastic material and bottom layer made up of pure isotropic material subjected to a pulsating axial force$P(t) = P_s + P_d cos\Omega t$ acting along its undeformed axis at one end, where P_s denotes the static component of load and P_d denotes the time-dependent component of load.

Where P_s and P_d are static component of load and time dependent component of load respectively and Ω denote disturbing frequency. As shown in the fig. 3 the element model comprises of two nodes. Each node has four degrees of freedom. Nodal displacements can be expressed as follows-

$$\{\ ^{e}\} = \{u_{1i}u_{3i}w_i\ \ _i u_{1j}u_{3j}w_j\ \ _j\} \quad (8)$$

Where i and j are nodal numbers for the element and $\{\Delta^e\}$ denote nodal displacement matrix of beam. The displacement in axial direction of the constraining layer, the displacement in transverse direction and the rotational angle can be written in terms of nodal displacements and finite element shape functions.

$$u_1 = [N_1]\{\ ^{e}\}, u_3 = [N_3]\{\ ^{e}\}, w = [N_w]\{\ ^{e}\},\ \ = [N_w]'\{\ ^{e}\}, \quad (9)$$

Where w and   are Transverse displacement and Rotational angle respectively.

The prime means differentiation with respect to axial co-ordinate 'x' and the shape functions are written as-

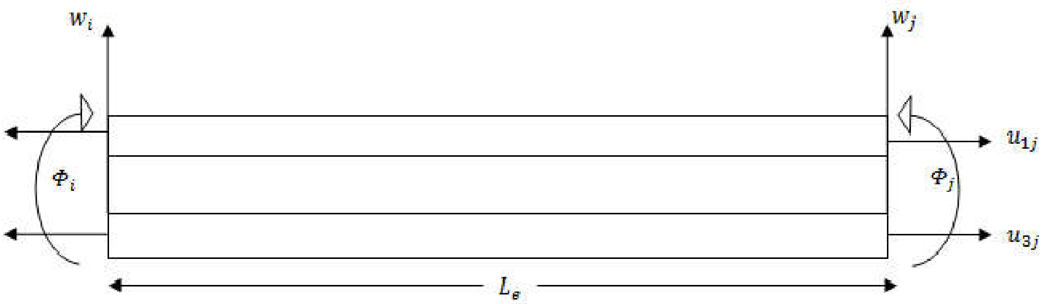
$[N_1] = [1\ \ \ \xi\ 0\ 0\ 0\ \xi\ 0\ 0\ 0]$

$[N_3] = [0\ 1\ \ \ \xi\ 0\ 0\ 0\ \xi\ 0\ 0]$

and

$$N_w = [0\ 0\ (1\ \ \ 3\xi^2 + 2\xi^3)(\xi\ \ \ 2\xi^2 + \xi^3)L_e\ 0\ 0\ 3\xi^2 + 2\xi^3(\ \ \xi^2 + \xi^3)L_e] \quad (10)$$

Where $\xi = {}^{X}/_{L_e}$ and L_e is the length of the element and N_w is Shape function matrix for transverse displacement.



Fig, 3. Sandwich beam element

RESULTS AND DISCUSSION

A three layered sandwich beam is considered. The materials taken for each layer are as follows-

- Top layer made up of FGM consisting of 100% ceramic S_3N_4 on its top face and 100% stainless steel SUS304 on its bottom face.
- Middle layer is viscoelastic.
- Bottom layer made up of stainless steel SUS304.

As shown (table.1) the temperature dependent coefficients of Young's modulus E (Pa), Poisson's ratio υ, co-efficient of thermal expansion α (1/K), mass density ρ (kg/m^3) and co-efficient of thermal conductivity $\hat{k}$ (W/mK) of SUS304 and Si_3N_4. The beam length and the breadth is taken as 0.6m &0.0254 m respectively. The thickness for top and bottom layer is taken as 0.03m. The thickness of viscoelastic core layer is taken as 0.5 times of thickness of top FGM layer initially.

The properties of top FGM layer are assumed to follow power law. The maximum and minimum temperatures of the top face of FGM layer are taken as 450K and 300K respectively. All other surfaces are assumed to be at room temperature i.e. 300K. The linear temperature variation along the thickness direction of FGM layer is plotted in figure (4).The temperature rise starts from bottom face of FGM layer at 300K and ends at top face at 450 K. The variation follows the same path for all power law indexes (n).

Table 1. Properties of ceramic and stainless steel

Material	P^T_{-1}	P^T_0	P^T_1	P^T_2	P^T_3	$P(at\ 300K)$
SUS304						
E	0	201.04×10^9	3.079×10^4	-6.534×10^{-7}	0	207.7877×10^9
Y	0	0.3262	-0.0002002	3.797*10^-7	0	0.3178
A	0	12.330*10^-6	8.086*10^-4	0	0	1.5321*10^-5
P	0	8166	0	0	0	8166
$\hat{k}$	0	12.04	0	0	0	12.04
Si_3N_4						
E	0	348.43*10^9	-0.000307	2.160*10^-7	-8.946*10^11	322.2715*10^9
N	0	0.24	0	0	0	0.24
A	0	5.8723*10^7	9.095*10^-4	0	0	7.4746*10^-6
P	0	2370	0	0	0	2370
$\hat{k}$	0	9.19	0	0	0	9.19

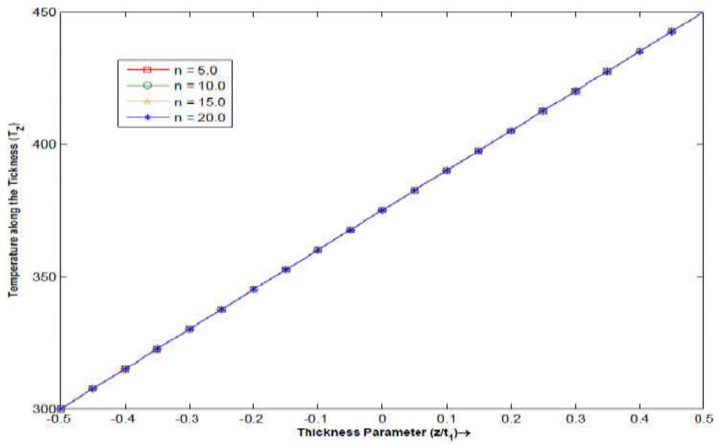


Fig. 4. Linear temperature variation along the thickness direction of FGM layer for different power index (n), T_t= 450 K and T_b= 300 K.

Free vibration analysis

For free vibration analysis, Frequency parameter (f) is calculated. Frequency parameter (f) is defined as the ratio of frequency of the sandwich beam to the frequency of an equivalent sandwich beam with pure stainless steel at the bottom layer and pure ceramic at the top layer.

Linear temperature variation

Figure (5) shows the variation of Frequency parameter (f) for the 1st three modes of vibration against core thickness parameter (t_2/t_1) for linear temperature variation. For power law index (n)=15.0 and temperature of top face of FGM layer is 450K. The core thickness parameter starts at 0.1 and ends at 3.0 with increment of 0.1. Here t_1 and t_2 are thickness of viscoelastic layer and stainless steel (SUS304) layer respectively.

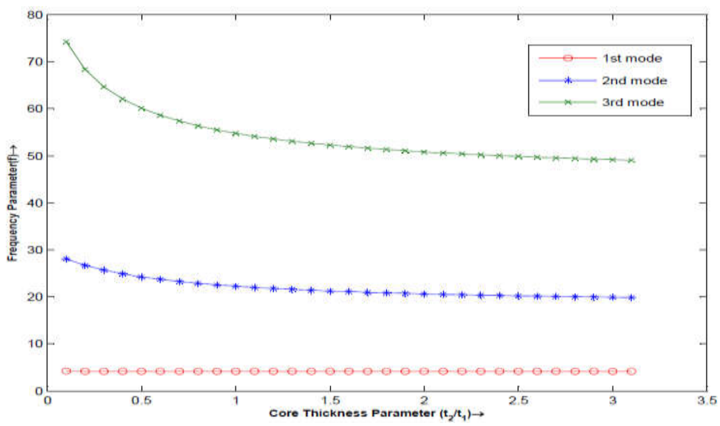


Fig. 5. Effect of core thickness parameter (t_2/t_1) on frequency parameter (f) for power low index(n)=15.0,η_c=0.18 T_t =450K and T_b =300K for linear temprature variation.

Non-linear temperature variation

Similarly, graphs are plotted for non-linear temperature variation under same conditions as stated in linear temperature variation case. Figure (6) shows the free vibration analysis for non-linear temperature vibration. The frequency parameter (f) varies in the same way as it did for linear temperature variation.

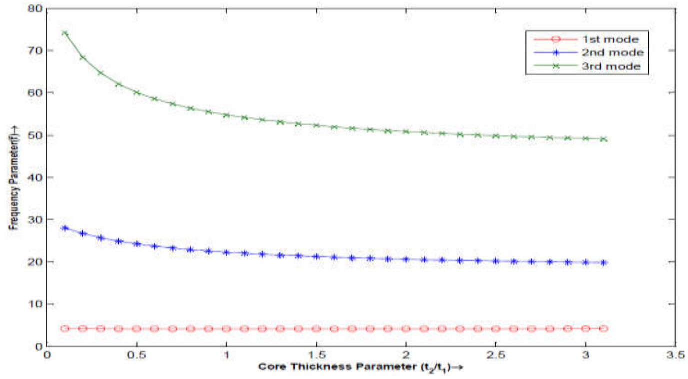


Fig. 6. Effect of core thickness parameter (t_2/t_1) on frequency parameter (f) for power low index(n)=15.0,η_c=0.18 T_t =450K and T_b =300K for non- linear temprature variation

Dynamic Stability Analysis

In Dynamic stability analysis, Dynamic Load Factor (β) is plotted against frequency ratio Ω/ω_o to get the instability regions for simple resonance. Here Ω is the external applied frequency and ω_o is the first natural frequency of the sandwich beam without temperature effect. The area inside the V-shape is the unstable region (U) and outside of it is stable region (S), as shown in figure (7). These instability regions are then compared by changing the values of core thickness parameter (t_2/t_1), power law index (n) and temperature of top face of FGM layer to understand the effect of system parameters on the instability of the system. It is found that with increase in power law index (n) from 5.0 to 15.0, the instability region shifts towards left i.e. it shifts towards lower frequency ratio or towards lower excitation frequency. This means that the probability of instability increases with increase in power law index (n).

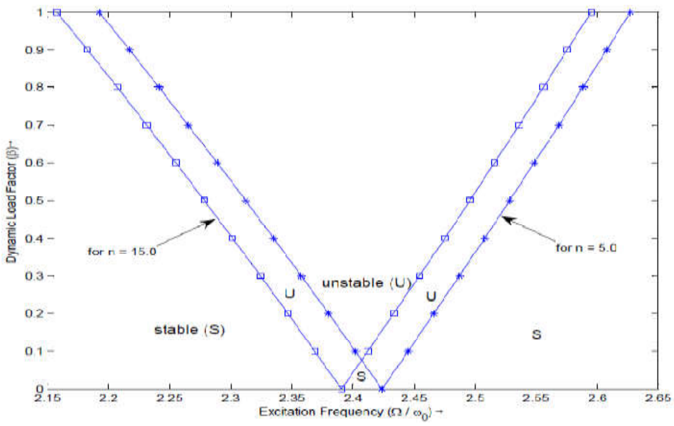


Fig. 7. Effect of power index(n) on instability region of the system forcore thickness parameter (t_2/t_1)=0.5, η_c=0.18 T_t =450K and T_b =300K for first mode of vibration,simple resonance

Conclusion

- The frequency of the beam decreases with increase in core thickness parameter (t_2/t_1) for 2ndand 3rdmode of vibration, keeping power law index (n) and top face temperature of FGM layer constant. For 1st mode, the frequency decreases up to the point at (t_2/t_1) = 1.5 and increases after that.
- The frequency of the beam decreases with increase in top face temperature of FGM layer, keeping power law index (n) and core thickness parameter constant.
- The instability of the system enhances with increase in core thickness parameter (t_2/t_1) for2ndand 3rdmode of vibration but for 1st mode of vibration, the instability of the system increases up to the point at (t_2/t_1) = 1.5 and decreases after that.
- The instability of the system enhances with increase in power law index (n).
- The instability of the system enhances with increase in top face temperature of FGM layer.

Future Scope of work

The present work can be extended to study the stability of structural elements like plates and shells.

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