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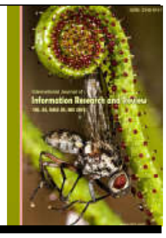

Research Article

VACUUM SOLUTION OF LRS BIANCHI TYPE-I MODEL IN *f(R)* THEORY OF GRAVITY

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ABSTRACT

In this paper we have obtained vacuum solution of locally rotationally symmetric (LRS) Bianchi type-I model in $f(R)$ theory of gravity assuming that the expansion θ is proportional to the shear scalar σ and hybrid scale factor $a=(t^k e^t)^{\frac{1}{n}}$ is used to find the deterministic solution of the model. Also physical behavior of the model are also discussed and the function of the Ricci scalar R is evaluated.

INTRODUCTION

Einstein's general theory of relativity explain the most of the gravitational phenomenon. But it fails to explain some of important problems in cosmology such as accelerating expansion of the universe. The present universe is dominated by dark energy causing cosmic acceleration and is supported by analysis of type I_a supernoma (SNI_a) (Perlmutter, 1997; Perlmutter, 1998; Perlmutter, 1999; Riess, 1998; Riess, 2004), cosmic microwave background anisotropic (Caldwell, 2002; Huang *et al*., 2006). To explain the accelerating expansion nature of the universe, many generalizations of Einstein's field equation have been proposed in the few decades. $f(T)$ theory of gravity is one of the example of generalized theory of gravity. This theory is a generalized version of teleparallel gravity in which the weitzenbock connection is used instead of Levi- Civita connection. Ratbay (2011) has shown that the acceleration of the universe understood by $f(T)$ gravity models. M. Sharif et.al (Sharif and Shamaila Rani, 2011) considered spatially homogeneous and anisotropic Bianchi type-I universe in $f(T)$ gravity theory. Wei H *et al* (2011) tried to constrain $f(T)$ theories with the fine structure constant. Bamba K. *et al* (2011) studied the cosmological evolution of the equation of state for dark energy with the combination of exponential, logarithmic and $f(T)$ theories. $f(R)$ theory of gravity is another example of generalized theory of gravity. This theory has attracted attention of the researcher in recent years. The $f(R)$ theory is actually an extension of standard Einstein-Hilbert action involving a function of the Ricci scalar R. Many authors have investigated $f(R)$ gravity in different context. Nojiri and Odinostsov (Nojiri and Odintsov, 2007; Nojiri and Odintsov, 2008) proved that the $f(R)$ theory of gravity provides very natural unification of the early time inflection and late time acceleration.

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Carrol *et al*. (2004) explain the presence of late time cosmic acceleration of the universe in $f(R)$ gravity. Bertolami *et al*. (2007) have proposed a generalization of $f(R)$ modified theory of gravity, by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m. Multamaki and Vilja (2006; Multam¨aki, 2007) investigated static spherically symmetric vacuum solution of the field equation and non vacuum solutions by taking fluid respectively. The universe seems to have an isotropic and homogeneous geometry at the end of inflammatory era (Linde, 2008). The anisotropy of the DE within the framework of Bianchi type space times is found to be usual in generating arbitrary ellipsoidality to the universe. It is believed that the early universe may not have been exactly uniform. In theoretical cosmology inhomogeneous and anisotropy model of the universe play an important role. The existence of anisotropic in early phases of universe is an interesting phenomenon to investigate. Bianchi type models are among simplest models with anisotropy background. Akarsu *et al.* (2010) have investigated Bianchi type –I anisotropic DE model with constant decelerating parameter. Anilkumar Yadav (20) studied some anisotropic dark energy models in Bianchi type-V space time. Yadav and Yadav (2011a), Yadav *et al.* (2011b) have studied anisotropic DE models with variable EoS parameter. Kumar and Singh (2007) solved the field equations in the presence of perfect fluid using a Bianchi type-I space time in general relativity. A bianchi type –III string cosmology with bulk viscosity has been studied by Xing –Xiang (2005). He assumed an expansion scalar proportional to the shear scalar to find the solution. Wang (2005) explored string cosmological models in Kantowski – Sachs space time. Vacuum and non vacuum solutions of Bianchi type –I and V space times in metric $f(R)$ gravity have been explored by M. Farsat Shamir (2009; Sharif, 2010). Sharif and Kausar (2011) investigated non vacuum solution of a Bianchi type –VI universe by considering the isotropic and anisotropic fluid as the source of dark matter and energy. Anirudhan Pradhan *et al.* (2012) discussed Bianchi –I massive string cosmological model in general relativity in which they assumed the law of variation of scale factor as increasing function of time as given by $a=(t^k e^t)^{\frac{1}{n}}$. M. Farsat Shamir (2015) studied locally rotationally symmetric (LRS) Bianchi type-I cosmology in $f(R,T)$ gravity. K.S. Adhav (2012) obtained LRS Bianchi type-I cosmological model in $f(R,T)$ theory of gravity. R .Venkateswarlu *et al.* (2013) studied LRS Bianchi type –III massive string cosmological models in scalar theory of gravitation. R. Venkateswarlu and J. Satish (2014) discussed LRS Biachi type –I in inflationary string cosmological model in Brans-Dicke theory of gravitation.

With this motivation, in this paper, we have obtained vacuum solution of locally rotationally symmetric (LRS) Bianchi type-I model in $f(R)$ theory of gravity with the assumption that the expansion θ is proportional to the shear scalar σ and hybrid scale factor $a=(t^k e^t)^{\frac{1}{n}}$ is used to find the deterministic solution of the model. Also physical behavior of the model are also discussed and the function of the Ricci scalar R is evaluated.

§ 2. Field equations in $f(R)$ theory of gravity

The $f(R)$ theory of gravity is the simplest generalization of the general theory of relativity proposed by Einstein in which Ricci scalar in Einstein–Hilbert action is replaced by an arbitrary function of the Ricci scalar. The action for $f(R)$ theory of gravity are given by

$$S=\int\left(\frac{1}{16\pi G}f(R)+L_m\right)\sqrt{-g}\; d^4x, \tag{1}$$

where $f(R)$ is general function of Ricci scalar R and L_m is the matter Lagrangian.

Now by varying the action S with respect to g_{ij} we obtain the field equations in $f(R)$ theory of gravity as

$$F(R)R_{ij}-\frac{1}{2}f(R)g_{ij}-\nabla_i\nabla_j F(R)+g_{ij}\,\Box F(R)=kT_{ij}, \tag{2}$$

where $F(R)\equiv\frac{df(R)}{dR}$, $\Box\equiv\nabla^i\nabla_i$, ∇_i is the covariant derivative and T_{ij} is the standard matter energy momentum tensor.

If we take $f(R)=R$, the field equations (2) reduce to the field equation of general theory of relativity which is propose by Einstein.

Contracting the above field equation (2), we have

$$F(R)R-2f(R)+3\,\Box F(R)=kT. \tag{3}$$

For vacuum, we have

$$F(R)R-2f(R)+3\Box F(R)=0 . \tag{4}$$

This gives an important relation between $F(R)$ and $f(R)$ which may be used to simplify the field equations and to evaluate $f(R)$.

From (4), we get

$$f(R)=\frac{1}{2}\left[3\Box F(R)+F(R)R\right] . \tag{5}$$

Using equations (2) and (5), the vacuum field equations take the form

$$\frac{F(R)R_{ij}-\nabla_i\nabla_j F(R)}{g_{ij}}=\frac{1}{4}[F(R)R-\Box F(R)] . \tag{6}$$

It follows that the equation (6) is not depend on the index i, hence above equation can be express as

$$A_i=\frac{F(R)R_{ij}-\nabla_i\nabla_j F(R)}{g_{ij}} . \tag{7}$$

and hence $A_i - A_j = 0$ for all i and j.

§ 3. LRS Bianchi Type-I Model

The line element of Bianchi type-I space-time is given by

$$ds^2=dt^2-A^2dx^2-B^2[dy^2+dz^2], \tag{8}$$

where A, B are called cosmic scale factors which are function of time t.

The corresponding Ricci scalar R becomes

$$R=-2\left[\frac{\ddot{A}}{A}+2\frac{\ddot{B}}{B}+2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}\right], \tag{9}$$

where (.) dot denotes derivative with respect to time t.

Thus, the subtraction of the 00-component and 11- component gives

$$-2\frac{\ddot{B}}{B}+2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{F}}{AF}-\frac{\ddot{F}}{F}=0 . \tag{10}$$

Similarly the subtraction of the 00-component and 22- component gives

$$-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{\dot{B}\dot{F}}{BF}-\frac{\ddot{F}}{F}=0 . \tag{11}$$

The average scale factor and the volume scale factor are defined respectively as under

$$a=(AB^2)^{\frac{1}{3}},\ V=a^3=AB^2 . \tag{12}$$

The generalized mean Hubble parameter H is defined by

$$H = \frac{1}{3}[H_1 + H_2 + H_3] = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a}, \tag{13}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = H_3 = \frac{\dot{B}}{B}$ are the directional Hubble parameters in the direction of x, y, z axes respectively.

The expansion scalar θ and shear scalar σ^2 are defined as follows

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B}, \tag{14}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right]^2, \tag{15}$$

where $$\sigma_{ij} = \frac{1}{2}(\nabla_j u_i + \nabla_i u_j) - \frac{1}{3}g_{ij}\theta. \tag{16}$$

The mean anisotropy parameter $\overline{A}$ is given by

$$\overline{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2\sigma^2}{3H^2}. \tag{17}$$

Solution of Field Equations

Equations (10) and (11) are two non-linear equations in three unknowns A, B and F. One additional constraint relating these parameters is used to obtain determinate solution of the system. So we used a physical condition that the expansion scalar θ is proportional to the shear scalar σ (i.e. $\theta \; \alpha \; \sigma$). This condition leads to the following relation between metric potentials:

$$A = B^n. \tag{18}$$

The subtraction the equation (11) and (10) gives

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} + \frac{\dot{A}\dot{F}}{AF} - \frac{\dot{B}\dot{F}}{BF} = 0. \tag{19}$$

After solving (19), we have

$$\frac{A}{B} = d_1 \exp\left[c_1 \int \frac{dt}{a^3 F}\right], \tag{20}$$

where d_1 and c_1 are constants of integration.

Following Anirudh Pradhan et. al. (2012), A. K. Yadav (2012; Yadav, 2012) we consider the generalized hybrid expansion law for scale factor as

$$a = (t^k e^t)^{\frac{1}{n}}. \tag{21}$$

The relation (21) gives combination of exponential law and power law expansion of the universe, which is Hybrid law expansion.In a recent paper Kotub Uddin *et al*. (2007) and Sharif and Shamir (2009) have established a result in the context of $f(R)$ gravity which show that

$$F \; \alpha \; a^m. \tag{22}$$

Thus using power law relation between F and a, we have

$$F = l\,a^{m}, \tag{23}$$

where l is the constant of proportionality, m is any integer (here taken as -2).

Using equations (21) and (23) for $m = -2$ in the equations (18) and (20), we obtained the scale factor as

$$A = \left[(1-n)c_1\, l^{\frac{3}{2}} \sum_{i=1}^{\infty} \frac{(2)^{i-1} t^{2k+i}}{(2k+i)(i-1)!} + c_2\right]^{\frac{n}{1-n}}. \tag{24}$$

$$B = \left[(1-n)c_1\, l^{\frac{3}{2}} \sum_{i=1}^{\infty} \frac{(2)^{i-1} t^{2k+i}}{(2k+i)(i-1)!} + c_2\right]^{\frac{1}{1-n}}, \tag{25}$$

From equations (24) and (25), the directional Hubble parameters in the directions of x, y and z axis are found to be

$$H_x = nc_1\, l^{\frac{3}{2}}\, t^{2k}\, e^{2t}\left[(1-n)\,c_1\, l^{\frac{3}{2}} \sum_{i=1}^{\infty} \frac{(2)^{i-1} t^{2k+i}}{(2k+i)(i-1)!} + c_2\right]^{-1} \tag{26}$$

and

$$H_y = H_z = c_1\, l^{\frac{3}{2}}\, t^{2k}\, e^{2t}\left[(1-n)\,c_1\, l^{\frac{3}{2}} \sum_{i=1}^{\infty} \frac{(2)^{i-1} t^{2k+i}}{(2k+i)(i-1)!} + c_2\right]^{-1}, \tag{27}$$

The mean Hubble parameter becomes

$$H = \frac{(n+2)}{3}\, c_1\, l^{\frac{3}{2}}\, t^{2k}\, e^{2t}\left[(1-n)\,c_1\, l^{\frac{3}{2}} \sum_{i=1}^{\infty} \frac{(2)^{i-1} t^{2k+i}}{(2k+i)(i-1)!} + c_2\right]^{-1}, \tag{28}$$

and volume scale factor is given by

$$V = \left[(1-n)\,c_1\, l^{\frac{3}{2}} \sum_{i=1}^{\infty} \frac{(2)^{i-1} t^{2k+i}}{(2k+i)(i-1)!} + c_2\right]^{\frac{3(n+2)}{1-n}}. \tag{29}$$

The expansion scalar θ and shear scalar σ^2 are given by

$$\theta = 3H = (n+2)\,c_1\, l^{\frac{3}{2}}\, t^{2k}\, e^{2t}\left[(1-n)\,c_1\, l^{\frac{3}{2}} \sum_{i=1}^{\infty} \frac{(2)^{i-1} t^{2k+i}}{(2k+i)(i-1)!} + c_2\right]^{-1}, \tag{30}$$

$$\sigma^2 = \frac{1}{3}(n-1)^2\left\{c_1\, l^{\frac{3}{2}}\, t^{2k}\, e^{2t}\left[(1-n)\,c_1\, l^{\frac{3}{2}} \sum_{i=1}^{\infty} \frac{(2)^{i-1} t^{2k+i}}{(2k+i)(i-1)!} + c_2\right]^{-1}\right\}^2. \tag{31}$$

The mean anisotropy parameter $\overline{A}$ turns out to be

$$\overline{A} = \left[\frac{2(n-1)}{(n+2)}\right]^2, \tag{32}$$

If $n = 1$ the model becomes isotropic, otherwise the model will be anisotropic. The mean anisotropic parameter in present model is independent of time this suggests that the anisotropy in expansion rates is maintained throughout the cosmic evolution.
The deceleration parameter q calculated as

$$q = \frac{nk}{(t+k)^2} - 1. \tag{33}$$

As at late phase of cosmic evolution i.e. $t \to \infty$, $q = -1$. We are concerned about late time acceleration expansion age of the universe so at late time q is a negative value which provided conformity with the recent observational data that our universe is an accelerating expansion stage.

The Ricci scalar for LRS Bianchi type-I model is given by

$$R = c_1\, l\, (t^k e^t)^2 \left[(1-n) c_1\, l^{\frac{3}{2}} \sum_{i=1}^{\infty} \frac{(2)^{i-1} t^{2k+i}}{(2k+i)(i-1)!} + c_2 \right]^{-1} \left\{ \begin{array}{l} c_1\, l^2\, (t^k e^t)^2 \left[(1-n) c_1\, l^{\frac{3}{2}} \sum_{i=1}^{\infty} \frac{(2)^{i-1} t^{2k+i}}{(2k+i)(i-1)!} + c_2 \right]^{-1} \\ \left[n(n+2)(1-2n) - 6 \right] - 2n(n+2) l^{\frac{1}{2}} (\frac{k+t}{t}) \end{array} \right\} \tag{34}$$

which clearly indicates that $f(R)$ cannot be explicitly written in terms of R. However, by inserting this value of R, $f(R)$ can be written as a function of t, which is true as R depends upon t.

$$f(R) = \frac{10cl(k+t)}{3t^{k+1}e^t} + 2\frac{l}{t^2}(t^k e^t)^{\frac{4}{3}} \left[\frac{4}{3}(k+t)^2 - k \right] - 3c_1 l^2 (t^k e^t)^{\frac{10}{3}} \left[(1-n) c_1 l^{\frac{3}{2}} \sum_{i=1}^{\infty} \frac{(2)^{i-1} t^{2k+i}}{(2k+i)(i-1)!} + c_2 \right]^{-1} \left\{ \begin{array}{l} \frac{27}{2} c_1 l^2 (t^k e^t)^2 \left[(1-n) c_1 l^{\frac{3}{2}} \sum_{i=1}^{\infty} \frac{(2)^{i-1} t^{2k+i}}{(2k+i)(i-1)!} + c_2 \right]^{-1} \\ + 5 l^{\frac{1}{2}} (\frac{k+t}{t}) \end{array} \right\}. \tag{35}$$

DISCUSSIONAND CONCLUSION

In this paper, we have studied the vacuum solution of LRS Bianchi–I model in $f(R)$ theory of gravity with time dependent deceleration parameter. We used the power law relation between $F(R)$ and a. We have also used shear scalar σ is proportional to scalar expansion θ which gives $A = B^n$, where A and B are the metric coefficient and n is an arbitrary constant.

- For this model, from equation (29),(30) and (27) it is clear that the expansion scalar θ, shear scalar σ and Hubble parameter H decrease with passage of time.
- The cosmological parameters H_x, H_y, H_z, θ and σ^2 are all infinite at $t = 0$.
- Volume scale factor V vanishes initially and increases with time. This shows that at the initial epoch, the universe starts expansion with zero volume and expands uniformly with increase of time.
- Equation (31) clearly indicates that the model becomes isotropic for $n = 1$, otherwise model will be anisotropic. Also the average anisotropic parameter is independent of time implying that anisotropy in expansion rates is maintained throughout the cosmic evolution.
- As $t \to \infty$, the value of decelerating parameter $q = -1$, this negative value of decelerating parameter indicates that the universe is in acceleration phase.

So we observed that the solutions obtained are quite new and certainly exhibit some interesting facts in a $f(R)$ theory of gravity.

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