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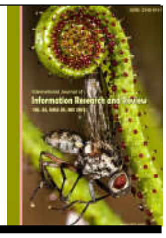

Research Article

TRIGONOMETRIC CURVE FITTING AND AN APPLICATION

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ABSTRACT

In this study, information was given about sinusoidal functions, that is to say, trigonometric functions. A trigonometric curve fitting was performed on the basis of a data set so that error sum of squares could be minimum. The ratio of air relative humidity (%) was investigated during the time period between 02.11.2012 at 00:00 PM and 06.11.2012 at 11:00 PM at Cebeci, Cankaya, Demetevler and Sincan districts of Ankara province. As a result of the analysis performed it was found that while the air humidity was lower during 09:00 AM -07: 00 PM. hours with high temperature; it was higher during 01:00 AM-07:00 AM hours with low temperature in general. It is expected that these results will recur periodically in the following periods. Therefore, a trigonometric curve can be fitted to these data.

INTRODUCTION

The periodic motions showing sinusoidal and cosine waves have widely been used in temperature and precipitation modelling in the fields of mathematics, physics, chemistry, astronomy, economics, geodesy, agriculture and other branches of science. The periodic functions used to study periodic phenomena such as oscillation and vibration movements are expressed by Fourier series (Altın 2011). Some issue-related studies were obtained as a result of a the literature search. Ogut (1998), in his study, was attempted to model daily traffic flows by using spectral analysis or Fourier analysis methods. In their study, Cetin et al. (1999) identified spatial variation structures of long term annually, monthly point precipitation observations of precipitation observation stations located in the Eastern Mediterranean Region, with geostatistical method. Mekik et al. (2005) performed measurements with 1 second intervals in times when traffic and heavy tonnage vehicles flows are busiest and most rare, in their studies named "Determination of the bridge oscillations and vibrations with real-time kinematic GP". In the consequence of the analysis, the variations of the oscillations and vibrations occured throughout the day were modeled by Fast Fourier Transform. Abbak (2005), in his study, analyzed sea level observations by spectral analysis with least squares method. Hourly observations of the year 1990 of Antalya and Mentes tide gauge stations were used for the analysis.

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As a result of the analysis, 5 hidden periodicities took place due to tides in the region of the stations were identified. Yurtcu ve Icaga (2005) calculated the periodic components of data of 5 well, 4 precipitation, 6 flow and 4 evaporation observation stations located in Akarcay basin, through well water level, flow, rainfall and evaporation observations. According to the results of periodogram analysis performed to investigate whether data are inclusive of any periodic behavior occurred throughout the year ; (2) periods were detected in the data of Seyitler (11001), Selevir (11004) precipitation stations; (1) period was detected in the data of Afyon (1034), Cay (11021) precipitation stations.

Two periods a year points out the change every four months, in other words, seasonal variation. Although it is not so clear in the periodogram analysis of the evaporation data, it can be said that three periodic components are available. One periodic component was observed in general in the data of flow and well water levels. In the study of Ustun (2008), forward business work simulation was applied firstly to Isıklı lake, considering the effects of climate change. The temperatures were modelled by Fourier series. Through the study performed, it has been suggested how climate change would be considered in planning of investment projects. It was revealed taking into account the increase trend of annual average air temperature in following periods that the effect of climate change on water resources, investment projects should be taken into account in the planning stage.The purpose of this study is fitting curve and performing an application on a data set by means of sinusoidal functions and trigonometric functions.

MATERIALS AND METHOD

The material of the study is consisted of relative humidity (%) values measured every hour between the period of 02.11.2012 at 00:00 PM and 06.11.2012 at 11:00 PM at Kecioren, Cebeci, Demetevler, Sıhhıye districts of Ankara province, acquired from the website of the Ministry of Environment and Urban Planning. Curve fitting was performed using these data, through trigonometric functions showing sinusoidal and cosine waves. Relative humidity is a measure obtained by division of the existing water vapor amount in the air to the water vapor amount that air could bear and is shown in percentage (http://www.cu.edu.tr/content/asp/turkish/cumeteosozluk.asp). The average relative humidity was considered in percentage and its average was calculated by summing up the values measured at 07:00 AM, 02:00 PM and 09:00 PM and then dividing by three (TSI, 2012). Lets assume that f(x) function is defined in the range of (-L, L) and with $f(x+2L) = f(x)$ out of this range, that is to say, let it have 2L period. Fourier series or Fourier expansion corresponding to f(x) is expressed as follows:

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right) \quad \text{......... (1)}$$

Here, a_0, a_n and b_n Fourier coefficients are as follows:

$$a_o = \frac{1}{L}\int_{-L}^{L} f(x)dx \quad \text{.......... (2)}$$

$$\begin{cases} a_n = \dfrac{1}{L}\displaystyle\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}dx \\ b_n = \dfrac{1}{L}\displaystyle\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}dx \end{cases} \quad n = 0,1,2,...$$

Fixed term in equation 2 equals to $\frac{a_0}{2} = \frac{1}{2L}\int_{-L}^{L} f(x)dx$. This is average of f(x) over a period. If $L = \pi$ then function has 2π period (Spiegel, 1963).

Considering these properties, any $f(x)$ function can be expanded to Fourier series as follows:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx) \quad \text{.......... (3)}$$

This kind of series is called as trigonometric series (Khuri, 2003). Taking term by term integral in the range of $[-\pi, \pi]$ over Fourier series and considering orthogonality relations given in equation 4; a_n and b_n coefficients are calculated as follows (Rudin, 1953):

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx \quad \text{(n=0,1,2,...)}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx \quad \text{(n=1,2,3,...)}$$

a_0 coefficient is :

$$a_o = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

Orthogonal functions are expressed as in Equation 4 (Presley, 1981)

$$\int_{-\pi}^{\pi} \cos nx \cos mx dx = 0, m \neq n$$

$$\int_{-\pi}^{\pi} \sin nx \cos mx dx = 0, \text{ all m, n} \quad \text{.............. (4)}$$

$$\int_{-\pi}^{\pi} \sin nx \sin mx dx = 0 \quad m \neq n$$

Fourier series convert a periodic function into the total of single waved functions (sine and cosine). Fourier series is a branch of Fourier analysis. Fourier series was used by Joseph Fourier for solving the metal bar or plate related heat equations between the dates 1768-1830. A heat equation is a fragmented differential equation. A general solution to this kind of heat equations wasn't available prior to Fourier's this study. Although fragmented approaches were available, they were insufficient. Because these approaches discussed the problem with the assumption that heat distribution is performed on the basis of simple equations. These basic / simple solutions were called as eigen solution. Fourier's idea was to create complex combinations of heat source by adding simple equations with (sine and cosine) coefficients successively. The sum of equations by specific coefficients is called Fourier series. Although this method firstly was applied for solution of heat problems, later on, it could be applied to the functions in very broad perspective. By using modern state of the theory, simple samples became more easy to understand. Fourier series have been used in many areas such as electrical engineering, vibration analysis, acoustic, signal processing, photo manipulation, quantum mechanics and economy calculations (Gallica - Fourier 1888).

Proximate expression of any $f(x)$ with Fourier series necessitates satisfaction of the necessary conditions. These conditions are monovalency of function at every point of its continuity in the range given, being finite and continuous in the range given, taking a finite number of maximum and minimum values in the range given (Bayram 2002). Under these conditions, Fourier series converge to f(x) at every point where function is continuous. Fourier series converge to arithmetic average of right and left function limits in discontinuity points of function (Buttkus 2000). In case $f(x)$ function is not continual and given as values in n equally spaced discontinuous points; total symbols are used instead of integral operations mentioned above. If the values of variable y show a periodic change with increasing of values of variable x, discontinuous Fourier series will be selected as the mathematical model. A periodic component created by effect of annual period in any parameter of series is determined by equation 5.

$$y_i = a_0 + \sum_{k=1}^{m}\left[a_k \cos\left(\frac{2\pi}{T}kx_i\right) + b_k \sin\left(\frac{2\pi}{T}kx_i\right)\right] \quad .... \quad (5)$$

Here y_i: The average value of the parameter, m: The number of significant harmonics, a_k, b_k: Fourier coefficients, T: period (Salas and Yevjevich, 1972). e_i residue (error) value in time i given in equation 5 (Bloomfield, 2000).

$$\theta_i = \frac{2\pi}{T}x_i, \quad i=1,2,,\ldots,n$$

When conversion is performed,

$$s = \sum_{i=1}^{n}\left\{y_i - \left[a_0 + \sum_{k=1}^{m}(a_k \cos k\theta_i + b_k \sin k\theta_i)\right]\right\}^2 \quad \ldots\ldots (6)$$

(Turker and Can 1997). Minimizing this expression

$$a_0 = \frac{1}{n}\sum_{i=1}^{n} y_i a_0 = \frac{1}{n}\sum_{i=1}^{n} y_i \,,$$

$$a_k = \frac{2}{n}\sum_{i=1}^{n} y_i \cos k\theta_i a_k = \frac{2}{n}\sum_{i=1}^{n} y_i \cos k\theta_i$$

And

$$b_k = \frac{2}{n}\sum_{i=1}^{n} y_i \sin k\theta_i b_k = \frac{2}{n}\sum_{i=1}^{n} y_i \sin k\theta_i$$

Formulas are obtained. Assuming n=1

Since $y_i = a_0 + a_1 \cos\theta_i + b_1 \sin\theta_i + e_i$,i=1,2,…,n

$$s = \sum_{i=1}^{n}\left[y_i - a_0 - a_1\cos\theta_i - b_1 \sin\theta_i + e_i\right]^2 \quad \ldots\ldots\ldots\ldots (7)$$

Can be written. When this expression is minimal, with partial differentiation according to the parameters A_0, A_1 and B_1 ;

$$na_0 + a_1\sum_{i=1}^{n}\cos\theta_i + b_1\sum_{i=1}^{n}\sin\theta_i = \sum_{i=1}^{n} y_i$$

$$a_0\sum_{i=1}^{n}\cos\theta_i + a_1\sum_{i=1}^{n}\cos^2\theta_i + b_1\sum_{i=1}^{n}\cos\theta_i\sin\theta_i = \sum_{i=1}^{n} y_i\cos\theta_i$$

$$a_0\sum_{i=1}^{n}\sin\theta_i + a_1\sum_{i=1}^{n}\sin\theta_i\cos\theta_i + b_1\sum_{i=1}^{n}\sin^2\theta_i = \sum_{i=1}^{n} y_i\sin\theta_i$$

are obtained. These equations can be written in a matrix form as below.

$$\begin{bmatrix} n & \sum\cos\theta_i & \sum\sin\theta_i \\ \sum\cos\theta_i & \sum\cos^2\theta_i & \sum\cos\theta_i\sin\theta_i \\ \sum\sin\theta_i & \sum\cos\theta_i\sin\theta_i & \sum\sin^2\theta_i \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum y_i\cos\theta_i \\ \sum y_i\sin\theta_i \end{bmatrix}$$

For sums in coefficients matrix,

$$\frac{1}{n}\sum_{i=1}^{n}\sin\theta_i \,,\ \frac{1}{n}\sum_{i=1}^{n}\cos\theta_i = 0 \ ;\ \frac{1}{n}\sum_{i=1}^{n}\sin^2\theta_i = \frac{1}{2}$$

$$\frac{1}{n}\sum_{i=1}^{n}\cos^2\theta_i = \frac{1}{2} \ ;\ \frac{1}{n}\sum_{i=1}^{n}\sin\theta_i\cos\theta_i = 0$$

solving above system;

$$\begin{bmatrix} a_0 \\ a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} 1/n & 0 & 0 \\ 0 & 2/n & 0 \\ 0 & 0 & 2/N \end{bmatrix}\begin{bmatrix} \sum y_i \\ \sum y_i\cos\theta_i \\ \sum y_i\sin\theta_i \end{bmatrix}$$

is obtained (Turker and Can, 1997).

RESULTS

Regarding average moisture ratio in the period between 02.11.2012 at 00:00 PM and 06.11.2012 at 11:00 PM, a curve fitting will be performed as below: The period of series is T=24.

$$y_i = a_0 + a_1\sin\theta_i + a_2\cos\theta_i \quad \ldots\ldots\ldots\ldots. (8)$$

$$\theta_i = \frac{2\pi}{24}t_i = \frac{\pi}{12}t_i \quad i=1,2,\ldots,24$$

Conversion is performed. Accordingly, using the values in Table 1, coefficients indicated in equation 8 are obtained.

According to these values,

For humidity ratio (%) regarding the values measured in Cebeci district of Ankara province;

$a_0 = 81,971$, $a_1 = 9,712$ and $a_2 = 12,693$ coefficients were obtained. By writing these coefficients in equation 8;

$$y_i = 81,971 + 9,712\sin\theta_i + 12,693\cos\theta_i$$

sinusoidal curve (trigonometric curve) was obtained. The graph of humidity values measured during day hours of the period in question is given in Figure 1.Similarly, curve was fitted to the relative humidity values (%) measured at the same date and hours using the trigonometric functions. For humidity ratio (%) regarding the values measured in Demetevler district of Ankara province;

$a_0 = 32,416$, $a_1 = 5,427$ and $a_2 = 8,956$ coefficients were obtained,

$$y_i = 32,416 + 5,427 \sin\theta_i + 8,956 \cos\theta_i$$

sinusoidal curve (trigonometric curve) was obtained. The graph of humidity values measured during day hours of the period in question is given in Figure 2. For humidity ratio (%) regarding the values measured in Kecioren district of Ankara province;

$a_0 = 55,901$, $a_1 = 10,714$ and $a_2 = 11,955$ coefficients were obtained,

$$y_i = 55,901 + 10,714 \sin\theta_i + 11,955 \cos\theta_i$$

sinusoidal curve (trigonometric curve) was obtained. The graph of humidity values measured during day hours of the period in question is given in Figure 3. For humidity ratio (%) regarding the values measured in Sıhhıye district of Ankara province;

$a_0 = 38,820$, $a_1 = 4,832$ ve $a_2 = 6,320$ coefficients were obtained.

$$y_i = 38,820 + 4,832 \sin\theta_i + 6,320 \cos\theta_i$$

Sinusoidal curve (trigonometric curve) was obtained. The graph of humidity values measured during day hours of the period in question is given in Figure 4. Because of that periodic functions used for expression of periodic phenomena are expressed with trigonometric series, that is to say, Fourier series (Altın, 2011), trigonometric curve (sinusoidal curve) was fitted. As a result of the analysis performed, the curve equations obtained separately for each district through trigonometric functions were given in Table 2.

Table 2. Trigonometric curves for relative humidity (%) values of the stations where measurements were performed

Measurement station	Trigonometric curve equation
Cebeci	$y_i = 81,971 + 9,712 \sin\theta_i + 12,693 \cos\theta_i$
Demetevler	$y_i = 32,416 + 5,427 \sin\theta_i + 8,956 \cos\theta_i$
Keçiören	$y_i = 55,901 + 10,714 \sin\theta_i + 11,955 \cos\theta_i$
Sıhhıye	$y_i = 38,820 + 4,832 \sin\theta_i + 6,320 \cos\theta_i$

Table 1. The sine and cosine values created according to t_i values

t_i	θ_i	$\sin\theta_i$	$\cos\theta_i$
0	0	0	1
1	$\pi/12$	0.258	0.966
2	$\pi/6$	0.499	0.867
3	$\pi/4$	0.705	0.709
4	$\pi/3$	0.864	0.503
5	$5\pi/12$	0.965	0.263
6	$\pi/2$	1.000	0.000
7	$7\pi/12$	0.967	-0.253
8	$2\pi/3$	0.869	-0.494
9	$3\pi/4$	0.712	-0.702
10	$5\pi/6$	0.507	-0.862
11	$11\pi/12$	0.268	-0.963
12	π	0.000	-1
13	$13\pi/12$	-0.248	-0.969
14	$7\pi/6$	-0.490	-0.872
15	$5\pi/4$	-0.698	-0.716
16	$4\pi/3$	-0.859	-0.512
17	$17\pi/12$	-0.962	-0.273
18	$3\pi/2$	-1.000	0.000
19	$19\pi/12$	-0.970	0.243
20	$5\pi/3$	-0.874	0.485
21	$7\pi/4$	-0.719	0.695
22	$11\pi/6$	-0.516	0.857
23	$23\pi/12$	-0.277	0.961

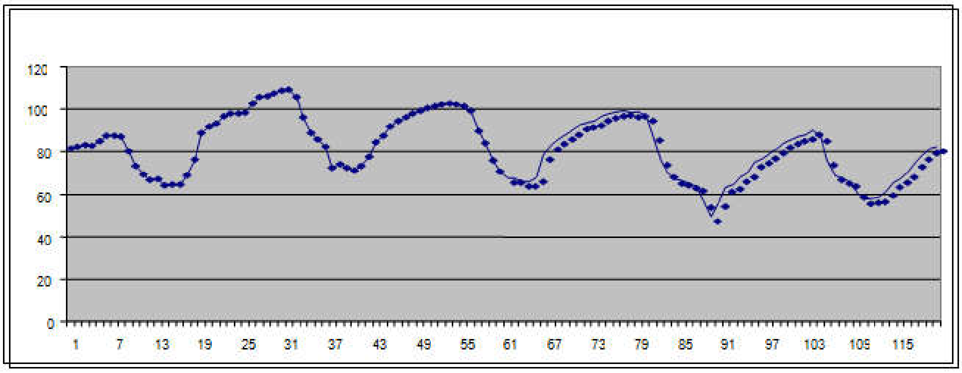

Figure 1. The relative humidity (%) values measured between the period of 02.11.2012 at 00:00 PM and 06.11.2012 at 11:00 PM at Cebeci district of Ankara province

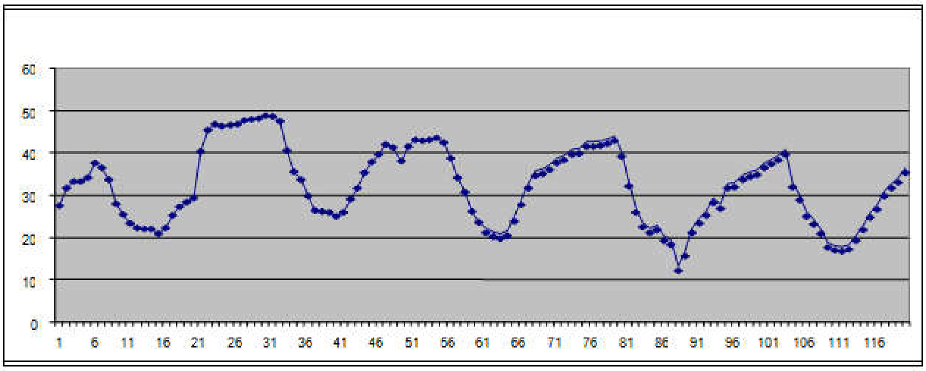

Figure 2. The relative humidity (%) values measured between the period of 02.11.2012 at 00:00 PM and 06.11.2012 at 11:00 PM at Demetevler district of Ankara province

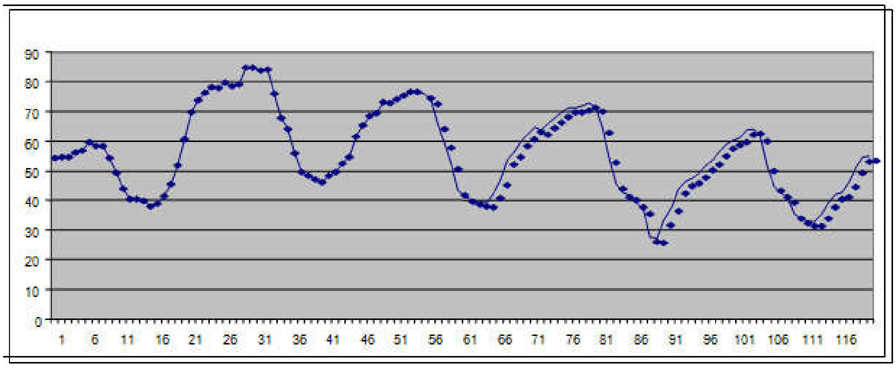

Figure 3. The relative humidity (%) values measured between the period of 02.11.2012 at 00:00 PM and 06.11.2012 at 11:00 PM at Kecioren district of Ankara province

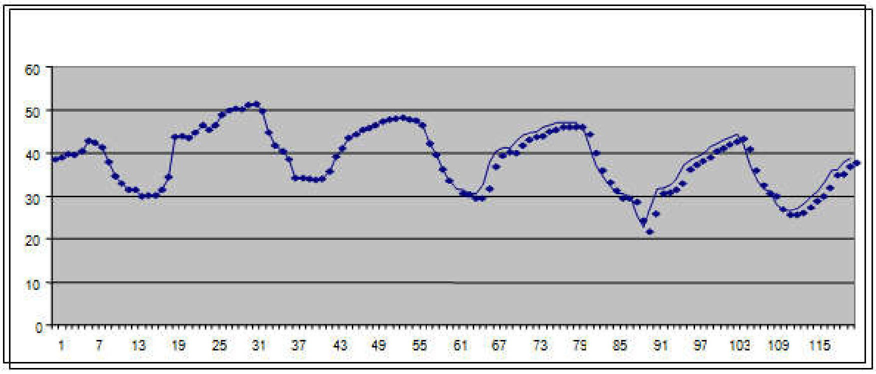

Figure 4. The relative humidity (%) values measured between the period of 02.11.2012 at 00:00 PM and 06.11.2012 at 11:00 PM at Sıhhıye district of Ankara province

Conclusion

In this study, the data were analyzed through Fourier series and theory and concepts were explained. The relative humidity (%) values measured every hour between the period of 02.11.2012 at 00:00 PM and 06.11.2012 at 11:00 PM at Kecioren Cebeci, Demetevler, Sıhhıye districts of Ankara province were examined in the study. As it is seen in Figure 1, the humidity ratio measured in Cebeci district of Ankara province ranges between 49.5-109. The humidity values were observed higher between the hours 00:00 PM -07:00 AM and lower between the hours 09:00 AM-07:00 PM. As it is seen in Figure 2, the humidity ratio measured in Demetevler district of Ankara province ranges between 13.3-48.8. The humidity values were observed higher between the hours 01:00 AM -07:00 AM and lower between the hours 09:00 AM-06:00 PM. As it is seen in Figure 3, the humidity ratio measured in Kecioren district of Ankara province ranges between 27.2-84.6. The humidity values were observed higher between the hours 10:00 PM - 07:00 AM and lower between the hours 10:00 AM-07:00 PM.
As it is seen in Figure 4, the humidity ratio measured in Sıhhıye district of Ankara province ranges between 22.8-51.5. The humidity values were observed higher between the hours 01:00 AM -07:00 AM and lower between the hours 00:00 AM-06:00 PM. Briefly, it was seen that while humidity ratio was lower at daylight hours with high temperature, it was lower at night hours with low temperature.

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