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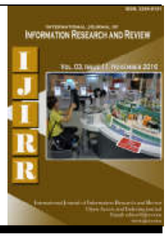

Research Article

ON COMPLETELY VAGUE Gα CONTINUOUS MAPPINGS

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ABSTRACT

The objective of this paper is to introduce vague completely generalized alpha continuous mapping in vague topological spaces. We investigate some of their properties and their inter relation between other continuous mappings are established with counter examples

INTRODUCTION

In this modern world the classical method of solving mathematics is not sufficient to solve our daily problems and our new requirements and hence some theories such as Fuzzy set theory (Zadeh, 1965), intuitionistic fuzzy set (Atanassov, 1986), soft set theory (Moiodtsov, 1999), Vague set theory (Gau and Buehrer, 1993) and so on has been developed to me*et al*l such problems. In this paper the concept of vague set is used in the vague topological spaces and studied the concept of vague completely generalized alpha continuous mappings.

2. Preliminaries

Definition 2.1: (Borumand and Zarandi, 2011**)** A vague set A in the universe of discourse U is characterized by two membership functions given by:

- A true membership function $t_A : U \to [0,1]$ and
- A false membership function $f_A : U \to [0,1]$

where $t_A(x)$ is a lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x", and $t_A(x) + f_A(x) \le 1$. Thus the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ 0f (0,1). This indicates that if the actual grade of membership of x is μ(x), then, $t_A(x) \le \mu(x) \le 1 - f_A(x)$. The vague set A is written as $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / u \in U\}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

Definition 2.2:(3) Let A and B be VSs of the form $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$ and $B = \{\langle x, [t_B(x), 1 - f_B(x)]\rangle / x \in X\}$ Then

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- $A \subseteq B$ if and only if $t_A(x) \le t_B(x)$ and $1 - f_A(x) \le 1 - f_B(x)$ for all $x \in X$
- A=B if and only if $A \subseteq B$ and $B \subseteq A$
- $A^c = \{\langle x, f_A(x), 1 - t_A(x)\rangle / x \in X\}$
- $A \cap B = \{\langle x, (t_A(x) \wedge t_B(x)), (1 - f_A(x) \wedge 1 - f_B(x))\rangle / x \in X\}$
- $A \cup B = \{\langle x, (t_A(x) \vee t_B(x)), (1 - f_A(x) \vee 1 - f_B(x))\rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, t_A, 1 - f_A\rangle$ instead of $A = \{\langle x, [t_A(x), 1 - f_A(x)]\rangle / x \in X\}$.

Definition 2.3: (Mariapresenti and Arockiarani, 2016**)** A vague topology (VT in short) on X is a family τ of VSs in X satisfying the following axioms.

- $0, 1 \in \tau$
- $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$
- $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a Vague topological space (VTS in short) and any VS in τ is known as a Vague open set(VOS in short) in X.

The complement A^c of a VOS A in a VTS (X, τ) is called a vague closed set (VCS in short) in X.

Definition 2.4:(Mariapresenti, 2016**)** A mapping $f : (X, \tau) \to (Y, \sigma)$ is called

- a vague continuous (V continuous in short) if $f^{-1}(B) \in VOS(X)$ for every $B \in \sigma$.
- a vague semi- continuous(VS continuous in short) if $f^{-1}(B) \in VSOS(X)$ for every $B \in \sigma$.
- a vague regular- continuous(VR continuous in short) if $f^{-1}(B) \in VROS(X)$ for every $B \in \sigma$.
- a vague pre-continuous(VP continuous in short) if $f^{-1}(B) \in VPOS(X)$ for every $B \in \sigma$
- a vague alpha continuous (Vα continuous in short) mapping if $f^{-1}(B)$ is a VαCS(X) in (X, τ) for every VCS B of (Y, σ).
- a vague irresolute (V irresolute in short) if $f^{-1}(B) \in VCS(X)$ for every VCS $B \in \sigma$.

Definition 2.5: (Mariapresenti, 2016**)** A mapping $f : (X, \tau) \to (Y, \sigma)$ is called

- a vague generalized continuous if $f^{-1}(B)$ is a VGCS in (X, τ) for every VCS B in (Y, σ).
- a vague generalized semi continuous if $f^{-1}(B)$ is a VGSCS in (X, τ) for every VCS B in (Y, σ).
- a vague generalized Pre continuous if $f^{-1}(B)$ is a VGPCS in (X, τ) for every VCS B in (Y, σ).
- a vague generalized irresolute if $f^{-1}(B)$ is a VGCS in (X, τ) for every VGCS B in (Y, σ).
- a vague alpha generalized continuous (VαG continuous in short) mapping if $f^{-1}(B)$ is a VαGCS(X) in (X, τ) for every VCS B of (Y, σ).
- a vague generalized alpha continuous (VGα continuous in short) mapping if $f^{-1}(B)$ is a VGαCS(X) in (X, τ) for every VCS B of (Y, σ).
- a vague generalized alpha irresolute (VGα irresolute in short) mapping if $f^{-1}(B)$ is a VGαCS in (X, τ) for every VGαCS B of (Y, σ).

Result 2.6: (Mariapresenti and Arockiarani, 2016**)** Every VCS, VGCS, VRCS, VαCS, is an VGαCS but the converses are not true in general.

Definition 2.7: (Mariapresenti, 2016**)** An VTS (X, τ) is said to be an vague $\alpha_k T_{1/2}$ space if every VGαCS in X is an VCS in X.

Definition 2.8: (Mariapresenti, 2016**)** Let (X, τ) be an VTS. The generalized alpha closure (Vgαcl(A) in short) for any A is defined as follows.

Vgαcl(A)=$\bigcap$ {K/K is an VGαCS in X and $A \subseteq K$ }. If A is an VGαCS, then Vgαcl(A)=A.

Remark 2.9: (Mariapresenti, 2016) It is clear that $A \subseteq Vg\alpha cl(A) \subseteq Vcl(A)$.

3. Vague completely generalized α continuous mappings

Definition 3.1: A mapping $f:(X,\tau)\to(Y,\sigma)$ is called a vague completely generalized alpha continuous (V completely Gα continuous in short) if $f^{-1}(B)$ is a VRCS in (X,τ) for every VGαCS B of (Y,σ).

Theorem 3.2

- Every Vague completely Gα continuous mapping is a VGα continuous mapping but not conversely.
- Every Vague completely Gα continuous mapping is a Vague continuous mapping but not conversely.
- Every Vague completely Gα continuous mapping is a Vα continuous mapping but not conversely.
- Every Vague completely Gα continuous mapping is a VαG continuous mapping but not conversely.
- Every Vague completely Gα continuous mapping is a VGS continuous mapping but not conversely.
- Every Vague completely Gα continuous mapping is a VGP continuous mapping but not conversely.
- Every Vague completely Gα continuous mapping is a VGα irresolute mapping, but the converse need not be true.

Proof: The proof of the above implications are obvious

The converse of the above theorem need not be true is given by the following example:

Example 3.3: Let X={a,b}, Y= {u,v} and $G_1=\{\langle x,[0.4,0.8],[0.3,0.8]\rangle\}$, $G_2=\{\langle x,[0.3,0.6],[0.1,0.7]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTS in X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)=u, f(b)=v. Then f is a VGα continuous mapping but not Vague completely Gα continuous mapping. Here G_2^c is a VGαCS in Y but not VRCS in X since $Vcl(V\text{int}(G_1^c))=0\neq(G_1^c)$.

Example 3.4: Let X={a,b}, Y= {u,v} and $G_1=\{\langle x,[0.5,0.8],[0.3,0.6]\rangle\}$, $G_2=\{\langle x,[0.3,0.6],[0.4,0.8]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTS in X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f (a)=u, f(b)=v. Then f is a vague continuous mapping but not V completely Gα continuous mapping.

Example 3.5: Let X= {a,b}, Y= {u,v} and $G_1=\{\langle x,[0.3,0.7],[0.4,0.7]\rangle\}$, $G_2=\{\langle x,[0.4,0.8],[0.5,0.7]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTS in X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)=u, f(b)=v. Then f is a Vα continuous mapping but not V completely Gα continuous mapping. Since G_2^c is a VGαCS in Y but not VRCS in X.

Example 3.6: Let X= {a,b}, Y= {u,v} and $G_1=\{\langle x,[0.3,0.8],[0.4,0.6]\rangle\}$, $G_2=\{\langle x,[0.1,0.5],[0.3,0.5]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTS in X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)=u, f(b)=v. Then f is a VαG continuous mapping but not V completely Gα continuous mapping. Since G_2^c is a VGαCS in Y but not VRCS in X. Since $Vcl(V\text{int}(G_1^c))=0\neq(G_1^c)$.

Example 3.7: Let X={a,b}, Y= {u,v} and $G_1=\{\langle x,[0.3,0.8],[0.4,0.7]\rangle\}$, $G_2=\{\langle x,[0.4,0.8],[0.5,0.7]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTS in X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)=u, f(b)=v. Then f is a VGS continuous mapping but not V completely Gα continuous mapping. Here G_2^c is a VGαCS in Y and since $Vcl(V\text{int}(G_1^c))=0\neq(G_1^c)$ it is not VRCS in X.

Example 3.8: Let X= {a,b}, Y= {u,v} and $G_1=\{\langle x,[0.3,0.7],[0.4,0.6]\rangle\}$, $G_2=\{\langle x,[0.2,0.8],[0.3,0.5]\rangle\}$. Then $\tau=\{0,G_1,1\}$ and $\sigma=\{0,G_2,1\}$ are VTS in X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f (a)=u, f(b)=v. Then f is a VGP continuous mapping but not V completely Gα continuous mapping. Here G_2^c is a VGαCS in Y and since $Vcl(V\text{int}(G_1^c))=0\neq(G_1^c)$ it is not VRCS in X.

Example 3.9: Let X={a,b}, Y= {u,v} and $G_1 = \{\langle x,[0.2,0.7],[0.4,0.6]\rangle\}$, $G_2 = \{\langle x,[0.1,0.5],[0.3,0.5]\rangle\}$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are VTS in X and Y respectively. Define a mapping $f:(X,\tau)\to(Y,\sigma)$ by f(a)=u, f(b)=v. Then f is a VGα irresolute mapping but not V completely Gα continuous mapping.

Definition 3.10: Let α, β∈ (0,1) and α+β≤1. A vague point (in short VP) p^x of X is a Vague set of X defined by p^x=<x, (t_p,1-f_p)>, for x∈X

$$t_p(y) = \begin{cases} \alpha \text{ if } & y = x \\ 0 \text{ if } & y \neq x \end{cases} \text{and}$$

$$1 - f_p(y) = \begin{cases} \beta \text{ if } & y = x \\ 0 \text{ if } & y \neq x \end{cases} \text{ and}$$

In this case, x is called the support of p^x. A Vague point p^x is said to belong to a VS A=<x, (t_A,1-f_A)> of X, defined by c(α,β)∈A, if α ≤ t_A(x) and β ≤ 1-f_A(x).

Definition 3.11: Let c (α,β) be a VP of a VTS (X,τ). A VS A of X is called a vague neighborhood (VN in short) of c(α,β) if there is a VOS B in X such that $c(\alpha,\beta) \in B \subseteq A$

Theorem 3.12: A mapping $f:(X,\tau)\to(Y,\sigma)$ is a Vague completely Gα continuous mapping if for every VP c(α,β)∈X and for every VN A of $f(c(\alpha,\beta))$, there exists a VROS B⊆ X such that $c(\alpha,\beta) \in B \subseteq f^{-1}(A)$.

Proof: Let c(α,β)∈X and let A be a VN of $f(c(\alpha,\beta))$. Then there exists a VOS U in Y such that $f(c(\alpha,\beta)) \in U \subseteq A$. Since every VOS is a VGαOS, U is a VGαOS in Y. Hence by hypothesis $f^{-1}(U)$ is a VROS in X and $c(\alpha,\beta) \in f^{-1}(U)$.Let B= $f^{-1}(U)$ Therefore $c(\alpha,\beta) \in B \subseteq f^{-1}(A)$.

Theorem 3.13: A mapping $f:(X,\tau)\to(Y,\sigma)$ is a Vague completely Gα continuous mapping then $Vcl(V\,\mathrm{int}(Vcl(f^{-1}(B)))) \subseteq f^{-1}(Vcl(B))$ for every VCS B in Y.

Proof: Let B be a VS in Y. Then Vcl (B) is a VCS in Y. Then Vcl(B) is a VGαCS in Y. By hypothesis $f^{-1}(Vcl(B))$ is a VRCS in X. Since every VRCS is VCS then $f^{-1}(Vcl(B))$ is a VCS in X. Therefore $Vcl(f^{-1}(Vcl(B))) = f^{-1}(Vcl(B))$. Now we have $Vcl(V\,\mathrm{int}(Vcl(f^{-1}(B)))) \subseteq Vcl(V\,\mathrm{int}(Vcl(f^{-1}(Vcl(B))))) \subseteq f^{-1}(Vcl(B))$

Theorem 3.14: Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping. Then the following are equivalent.

- f is a vague completely Gα continuous mapping
- $f^{-1}(B)$ is a VROS in X for every VGαOS B in Y.
- For every VP c(α,β) ∈X and for every VGαOS B in Y such that if $f(c(\alpha,\beta)) \in B$ there exists a VROS A in X such that c(α,β) ∈A and f(A) ⊆B.

Proof: (i)⇒(ii) is obvious.

- ⇒ Let c(α,β) ∈ X. Let B be a VGαOS in Y and $f^{-1}(B)$ is a VROS in X. Let $f(c(\alpha,\beta)) \in B$ and let A= $f^{-1}(B)$. Then $c(\alpha,\beta) \in f^{-1}(f(c(\alpha,\beta))) \in f^{-1}(B) = A$. Therefore c(α,β) ∈ A and $f(A) = f(f^{-1}(B)) \subseteq B$. This implies f(A)⊆B.
- ⇒(i) Let B be a VGαOS in Y and let c(α,β) ∈ A and f(c(α,β))∈B. Then by hypothesis there exists a VROS G such that c(α,β)∈G and f(G)⊆B. Now c(α,β)∈f^{-1}(B). But G⊆f^{-1}(B), That is $f^{-1}(B) = \bigcup_{c(\alpha,\beta)\in f^{-1}(B)} G \subseteq f^{-1}(B)$. This implies

$f^{-1}(B) = \bigcup_{c(\alpha,\beta)\in f^{-1}(B)} G$ where G is a VROS and hence $f^{-1}(B)$ is a VROS in X. Hence $f^{-1}(B)$ is a vague completely Gα continuous mapping.

Theorem 3.15: A mapping $f:(X,\tau)\to(Y,\sigma)$ is a Vague completely Gα continuous if and only if $f^{-1}(A)$ is a VROS in X for every VGαOS A in Y.

Proof: Necessity: Let A be a VGαOS in Y. This implies A^c is a VGαCS in Y. Since f is a vague completely Gα continuous mapping, $f^{-1}(A^c)$ is a VRCS in X. Hence $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a VROS in X.

Sufficiency: Let A be a VGαCS in X. This implies A^c is a VGαOS in Y. By hypothesis $f^{-1}(A^c)$ is a VROS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a VRCS in X. Hence f is a vague completely Gα continuous mapping.

Theorem 3.16: Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\Omega)$ be any two mappings. Then the following statements hold:

- f be a vague completely Gα continuous mapping and g be a vague continuous mapping. Then their composition $g\circ f:(X,\tau)\to(Z,\Omega)$ is a vague completely Gα continuous mapping
- f be a vague completely Gα continuous mapping and g be a vague α continuous mapping. Then their composition $g\circ f:(X,\tau)\to(Z,\Omega)$ is a vague completely Gα continuous mapping
- f be a vague completely Gα continuous mapping and g be a vague completely continuous mapping. Then their composition $g\circ f:(X,\tau)\to(Z,\Omega)$ is a vague completely Gα continuous mapping
- f be a vague completely Gα continuous mapping and g be a vague Gα irresolute mapping. Then their composition $g\circ f:(X,\tau)\to(Z,\Omega)$ is a vague completely Gα continuous mapping
- f be a vague completely Gα continuous mapping and g be a vague Gα continuous mapping. Then their composition $g\circ f:(X,\tau)\to(Z,\Omega)$ is a vague completely Gα continuous mapping

Proof: (i) Let A be a VCS in Z. Then $g^{-1}(A)$ is a VCS in Y, By hypothesis. Since every VCS is a VGαCS, $g^{-1}(A)$ is a VGαCS in Y. Therefore $f^{-1}(g^{-1}(A))$ is a VRCS in X. Hence $g\circ f$ is a V completely Gα continuous mapping.

The proof of (ii), (iii),(iv) and (v) are similar to (i).

Theorem 3.17: Let $f:(X,\tau)\to(Y,\sigma)$ be a vague completely Gα continuous mapping. Then the following statements hold.

- $f(Vg\alpha cl(A)) \subseteq Vcl(f(A))$, for every VS A in X.
- $Vg\alpha cl(f^{-1}(B)) \subseteq f^{-1}(Vcl(B))$, for every VS B in X.

Proof: (i) Let A $\subseteq$X. Then Vcl (f(A)) is a VCS in Y. Since f is a vague completely Gα continuous mapping, $f^{-1}(Vcl(f(A)))$ is a VGαCS in X. Since $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(Vcl(f(A)))$ and $f^{-1}(Vcl(f(A)))$ is a VGα closed, implies $Vg\alpha cl(A) \subseteq f^{-1}(Vcl(f(A)))$. Hence $f(Vg\alpha cl(A)) \subseteq Vcl(f(A))$.

(ii)Replacing A by $f^{-1}(B)$ in (i), we get $f(Vg\alpha cl(f^{-1}(B))) \subseteq Vcl(f(f^{-1}(B))) \subseteq Vcl(B)$ Here $Vg\alpha cl(f^{-1}(B)) \subseteq f^{-1}(Vcl(B))$ for every VS B in Y.

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