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Research Article

OPTIMAL ORDERING POLICY FOR DETERIORATING ITEMS WITH POWER-FORM STOCK DEPENDENT DEMAND AND SHORTAGES UNDER TWO-WAREHOUSE FACILITY

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ABSTRACT

In realistic world, there usually exist various factors that induced the retailer to order more items than the capacity of his own warehouse (OW). Therefore, for the retailer, it is very practical to determine whether or not to rent other warehouse and what policy to adopt if other warehouse is intended needed. For the stock dependent demand pattern, retailer has his own warehouse to display the item and may another warehouse of the larger capacity, treated as rented warehouse (RW) to storages the excess inventory. In this paper, an inventory model with power form stock dependent demand rate is developed. The demand rate is assumed to be a polynomial form of initial inventory level in OW. It also assumed that retailer first fulfills the demand (depending upon the stock displayed in the OW) directly from the RW until the inventory level in the RW reaches to the zero level, after that , demand is fulfilled from OW. As a consequence, no item is transferred from RW to OW, therefore no transfer cost (neither fixed nor variable) is considered between RW and OW. It is considered that the deterioration rate per unit items in the RW and OW are different due to different preservation environments, as a consequence the holding costs per unit in RW and OW are also different. Proposed model is illustrated with some numerical examples along with some sensitivity analysis of parameters using LINDO software.

INTRODUCTION

The classical inventory models are mainly developed with the single storage facility without capacity constraint. But, thinking in more practical terms, in the busy markets like supermarket, corporation market, and municipalities market etc., the storage area of the items is limited. In some practical situations, when suppliers provide price discounts for bulk purchases or when the items are under consideration is seasonal product such as the output of harvest or the procuring goods is higher than the other inventory related cost or demand of items is very high or there are some problems in procurement, retailer may decide to purchase a large amount of items at a time than can be storage in its own warehouse (OW). In this situation for storing the excess items, an additional storage space is often required. In this case, retailer may either rent other warehouse or rebuild a new warehouse. However from economic point of views, they usually choose to rent other warehouse known as rented warehouse (RW), which is assumed to be available with abundant capacity.

The inventory costs (including holding cost and deteriorating cost) in RW are usually higher than those in OW due to additional cost maintenance, materials handling, etc. and hence the items are stored first in OW and only excess stock is stored in RW. To reduce the inventory costs, it will be economical to consume the goods of RW at the earliest. An early discussion on the effect of two-warehouses was considered by Hartely (Hartley, 1976). Recently other authors have considered this type of inventory model. Sarma (Sharma, 1987) developed a deterministic inventory model with finite replenishment rate. Dave (Dave, 1988) further discussed the cases of bulk release pattern for both finite and infinite replenishment rates. Pakkala and Achary (1992) extended the two-warehouse inventory model for deteriorating items with finite replenishment rate and shortages, taking time as discrete and continuous variable, respectively. Bhunia and Maiti (1998) considered a two-warehouse inventory model for deteriorating items with a linear trend in demand and shortages. Kar *et al*. (2001) assumed deterministic inventory model with two levels of storage, a linear trend in demand and a fixed time horizon. Yang (2006) developed two-warehouse partial backlogging inventory model. Dye *et al*. (2008) developed a deterministic inventory model for deteriorating items with capacity constraint and time-proportional back logging rate. Hsieh *et al*. (2008) determine optimal lot size for a two-warehouse system.

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Vishnoi and Shon (2010) discussed an inventory model for non-instantaneous deteriorating items for two levels of storage. Ghosh *et al*. (2011) developed an inventory model for a deteriorating item with two levels of storage where the demand rate is a function of the on-hand inventory. The stock is transported periodically in bulk from RW to OW and there is an associated transportation cost. Shortages are not allowed and an algorithm is developed to obtain the optimal solution of the model. It is well known that the stock level has a motivational effect on the customers in a supermarket; i.e. the demand rate may go up or down if the on-hand inventory level increases or decreases. In corporate world such a situation is known as the stock-dependent demand. Datta and Pal (1990) discussed an infinite time-horizon deterministic inventory model without shortages, where the demand rate at any instant depends on the on-hand inventory (stock level) at that instant down to a certain stock level and then it becomes constant for the remaining period of the cycle i.e. $D(t)=\alpha[Q(t)]^{\beta} \; ,if\, Q(t)>S_0$ and $D(t)=\alpha S_0^{\beta} \; ,if\, 0\le Q(t)\le S_0$. Recently a large number of mathematical models have been reported in the existing literature. Among them, to get the idea of the trends of recent research, one may refer to the works of Datta *et al*. (1998), Chang and Dye (1999), Balkhi and Benkherouf (2004), Balakrishnan *et al*. (2004), Teng *et al*. (2005) extend Datta and Pal (1990)'s model to allow for not only deteriorating items but also non-zero ending inventory. Inventory problem considered three possible cases,

- Initial stock-level (q) is less than or equal to S_0 (i.e., $q\le S_0$),
- (b) Initial stock-level (q) is higher than S_0 and S_0 is higher than the ending inventory level i_T (i.e., $q>S_0>i_T$),
- Ending inventory i_T is higher than or equal to S_0 (i.e. $q>i_T\ge S_0$), and establish the necessary and sufficient conditions for each case. They proposed an algorithm to determine the optimal replenishment cycle time and ordering quantity such that the total profit per unit time is maximized.

Teng and Chang (2005) determined economic production quantity model for deteriorating items with price and stock dependent demand. Wu *et al*. (19) obtained an optimal replenishment policy for non instantaneous deteriorating items with stock-dependent demand and partial backlogging. Singh *et al*. (2007) determined optimal ordering policy for decaying items with stock-dependent demand under inflation. Sajadieh *et al*. (2010) developed an integrated vendor-buyer model with stock-dependent demand. Deterioration of physical goods is one of the important factors in any inventory and production system. It is important to control and maintain the inventories of decaying items for the modern corporation. Most of the physical goods undergo decay or deterioration over time. Commodities such as fruits, vegetables and foodstuffs suffer from depletion by direct spoilage while kept in store. Highly volatile liquids such as alcohol, gasoline, etc. undergo physical depletion over time through the process of evaporation. Electronic goods, photographic film, grain, chemicals, pharmaceuticals etc. deteriorate through a gradual loss of potential or utility with the passage of time. Thus deterioration of physical goods in stock is very realistic feature. Ghare and Schrader (1963) first developed a model with constant deterioration rate. Dave and Patel (1981), Deb and Chaudhuri (1986), discussed different types of inventory models for items with constant deterioration rate. Roy *et al*. (2009) developed an inventory model for a deteriorating item over a random planning horizon. Singh and Singh (2010) developed two echelon supply chain model with imperfect production for weibull distribution deteriorating items.

In this paper, we develop two-warehouse model with power-form stock-dependent demand and shortages under the assumption that the deterioration rate per unit items in the RW and OW are different due to different preservation environments. Items are not transferred from RW to OW; rather, Retailer first fulfills the demand (depending upon the stock displayed in the OW) directly from the RW until the inventory level in the RW reaches to the zero level after that, demand is filled from OW up to the time inventory level in OW reaches to the zero level. The purpose of this paper is to determine whether, the retailer should hire the rented warehouse or not, under the different circumstances.

Assumptions and notations

The notations adopted in this paper are as follows

c	the purchasing cost per unit item
c_h	the holding cost per unit per unit time in OW
c_{hr}	the holding cost per unit per unit time in RW
c_d	the deterioration cost per unit deteriorated item
c_s	the shortage cost per unit item
θ	deterioration rate of the inventory
F	the fixed replenishment cost per replenishment for a single warehouse system
F_1	the fixed replenishment cost per replenishment for a two-warehouse system
W	the storage capacity of OW
T	the length of replenishment cycle
q	the replenishment quantity per replenishment
t_r	time at which the inventory in the RW reaches to zero level.
t_1	time at which the shortage starts
Q(t)	the inventory level at time t
D(t)	the demand rate per unit time
TP_1	total profit function for single-warehouse
TP_2	total profit function for two-warehouse

The following assumptions are used in developing the model

(1). Replenishment is instantaneous.
(2). the time horizon of the inventory system is infinite.
(3). the demand rate, D(t), is assumed to be dependent on the initial inventory level and of polynomial form-that is to say, D (t) $=\alpha(Q_i(t))^\beta$.
(4). Shortages are allowed to occur.
(5). the rented warehouse RW has unlimited capacity.
(6). we are well aware of the storage condition in OW, but we are not confidently sure about the storage conditions in RW. Therefore to more matching the problem to realistic situation it is assumed that the preservation conditions in both warehouses are different. As a consequence it is assumed that the items in the OW follow a deterioration rate θ whereas the items in RW follow weibull distribution deterioration i.e. items deteriorates at the rate $\gamma\delta\, t^{\delta-1}$, where γ is the scale parameter and δ is the shape parameter. Note: if we take θ=γ and δ=1, then the deterioration rate in both warehouses are same.
(7). Due to different preservation environments, holding costs per unit item in RW and OW are different as a consequence it is assumed that the holding cost per unit item in RW is higher than that of OW.
(8). Items are not transferred from RW to OW instead; initially demand is fulfill directly from the RW until the inventory level in the RW reaches to the zero level at the time t_r after that, demand is fulfilled from OW up to the time t_1 at which inventory level in OW, reaches to the zero level.

Our problem is to be discussed in this paper are

(a) How the decision-maker knows whether or not to rent RW to hold more items under the situation defined above;
(b) What order lot-size the decision-maker should make if he needs indeed to rent RW.

For answering the first question, we first simply depict the single warehouse system.

Single warehouse models

The inventory system with a single warehouse can be stated as follows: The retailer ordered q quantities initially in each of the replenishment. The inventory level decreases due to the combined effect of the demand and the deterioration in the interval (0, T).

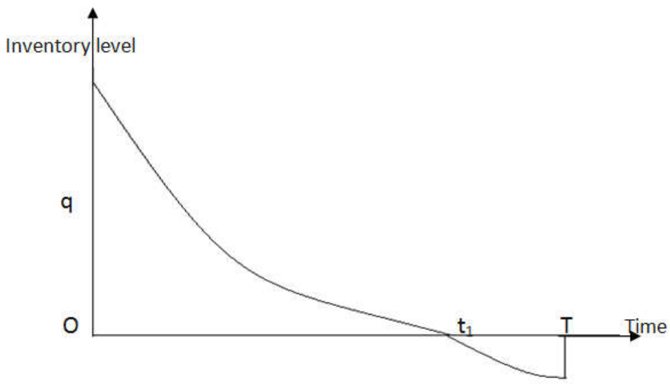

Fig 1. Retailer's inventory level Inventory Level in Single-warehouse for single warehouse

Differential equations governing the model are as follows:

$$\frac{dQ(t)}{dt} + \theta Q(t) = -\alpha[Q_i(t)]^\beta \quad , \quad 0 \le t \le t_1 \tag{1}$$

and

$$\frac{dQ(t)}{dt} = -\alpha[Q_i(t)]^\beta \quad , \quad t_1 \le t \le T \tag{2}$$

With Boundary condition Q(0) =q = Q_i(t) and Q(t_1) =0, on solving the above equations, we have

$$Q(t) = \left(q + \frac{\alpha}{\theta} q^\beta\right) e^{-\theta t^2/2} - \frac{\alpha}{\theta} q^\beta \quad , \quad 0 \le t \le t_1 \tag{3}$$

and

$$Q(t) = \alpha q^{\beta} (t_1 - t) \quad , \qquad t_1 \le t \le T \tag{4}$$

At, t= t_1 & Q (t_1) =0 Equation (3) gives

$$\left(q + \frac{\alpha}{\theta} q^{\beta}\right) - \frac{\alpha}{\theta} q^{\beta} e^{\theta t_1} = 0 \tag{5}$$

Now, the different costs corresponding to the single warehouse model which we consider here first are follows as:

Holding Cost: The holding costs should include that of all produced items, Defective and non-defective. The holding cost consists during the interval (0,t_1) and is given by

$$HC = C_h \int_0^{t_1} Q(t)\, dt$$

$$= C_h \int_0^{t_1} . \left\{ \left(q + \frac{\alpha}{\theta} q^{\beta}\right) e^{-\theta t_1} - \frac{\alpha}{\theta} q^{\beta} \right\} dt$$

$$= \frac{c_h}{\theta} \left\{ \left(q + \frac{\alpha}{\theta} q^{\beta}\right) \left(1 - e^{-\theta t_1}\right) - \alpha q^{\beta} t_1 \right\} \tag{6}$$

Deterioration Cost: The deterioration cost of the model consists during the interval (0,t_1) and is given by

$$DC = C_d \int_0^{t_1} \theta . Q(t)\, dt$$

$$= C_d \int_0^{t_1} \theta \left\{ \left(q + \frac{\alpha}{\theta} q^{\beta}\right) e^{-\theta t_1} - \frac{\alpha}{\theta} q^{\beta} \right\} dt$$

$$= C_d \left[\left(q + \frac{\alpha}{\theta} q^{\beta}\right) \left(1 - e^{-\theta t_1}\right) - \alpha q^{\beta} t_1 \right] \tag{7}$$

Shortages Cost: The shortages cost consists during the time (t_1, T) is given by

$$SC = C_s \int_{t_1}^{T} Q(t)\, dt$$

$$= C_s\, \alpha q^{\beta} \int_{t_1}^{T} (t_1 - t) dt$$

$$= C_s\, \alpha\, q^{\beta} \left\{ T t_1 - \frac{1}{2}\left(T^2 + t^2\right) \right\} \tag{8}$$

Purchasing cost

PC = c q (9)

Fixed cost: The fixed replenishment cost per replenishment for a single warehouse system = F (10)

Total cost of Single warehouse system

Total cost govern by above all the costs

$TC_1(q)$ = Holding cost + Cost of Defective Items + Shortage cost + Purchasing Cost+Fixed cost

$$= \frac{c_h}{\theta}\left\{\left(q+\frac{\alpha}{\theta}q^{\beta}\right)\left(1-e^{-\theta t_1}\right)-\alpha q^{\beta} t_1\right\} + C_d\left[\left(q+\frac{\alpha}{\theta}q^{\beta}\right)\left(1-e^{-\theta t_1}\right)-\alpha q^{\beta} t_1\right]$$

$$+ C_s\,\alpha\, q^{\beta}\left\{T t_1-\frac{1}{2}\left(T^2+t^2\right)\right\} + c\,q + F \quad (11)$$

From Eq. (11), it is clear that the cost function is a function of single variable q. Thus, from the necessary condition for $TC_1(q)$ to be minimum, the optimal value of q satisfies $\frac{dTC_1(q)}{dq}=0$, gives

$$\left(\frac{C_h}{\theta}+C_d\right)\left\{\left(1+\frac{\alpha\beta}{\theta}q^{\beta-1}\right)\left(1-e^{-\theta t_1}\right)-\alpha\beta t_1\, q^{\beta-1}\right\} + \alpha\beta C_s\, q^{\beta-1}\left\{T t_1-\frac{1}{2}\left(T^2+t^2\right)\right\}+C=0 \quad (12)$$

The optimal replenishment quantity q*, can be obtained on solving the Eq. 12, Now the optimal replenishment quantity q* may be either greater than or less than W. if q*<W, it implies that the retailer does not obtain more benefits from ordering more than q* units. In other words, retailer does not need the RW to enlarge his ordering quantity. In contrast, the retailer will need to consider the ordering problem of the two-warehouse system because holding more items than W units by using RW in this situation may bring more benefits to the retailer. The following theorem shows the necessary and sufficient condition of judging whether q* is less than W. this make the retailer to very easily determine whether or not to take the two warehouse system into account.

Theorem 1: The optimal order quantity q* of the single-warehouse system is no less than W if and only if M≥0, where

$$M=\alpha\beta W^{\beta-1}\left[\frac{\left(C_h+\theta C_d\right)\left\{\left(1-e^{-\theta t_1}\right)-\theta^2 t_1\right\}}{\theta^2}+C_s\left\{T t_1-\frac{1}{2}\left(T^2+t^2\right)\right\}\right]+c \quad (12a)$$

The above theorem illustrates that if M<0 the retailer uses only OW and orders q* units, whereas the retailer needs to know the optimal order policy of the two-warehouse system as M≥0.

In the next section, we develop the mathematical model for the two-warehouse system to find the optimal ordering policy for the retailer.

Two-warehouse model

In the two-warehouse model retailer order q units in each replenishment cycle, out of which W units are kept in OW and the remaining (q – W) units are kept in the RW. Retailer displayed the stock in OW to attracts the customers and fulfill the demand from the RW until the inventory level in the RW reaches to the zero level at the time "t_r", after that, demand is filled from OW up to the time "t_1" at which inventory level in OW, reaches to the zero level.

Model formulation for the time period, $0 \le t \le t_r$

In the time period $0 \le t \le t_r$, since the demand in this duration is being fulfilled from the RW therefore in this time period, the inventory level in the own-warehouse decreases due to the deterioration only. Hence the differential equations governing the model is as follows

$$\frac{dQ_0(t)}{dt}=-\theta\, Q_o(t)\ ,\quad 0\le t\le t_r \quad (13)$$

With the boundary condition $Q_0(0) = W$.

Therefore, the inventory level at any time t in the OW is given by solving the eqn. 13, we get

$$Q_0(t) = W e^{-\theta t}\ ,\quad 0\le t\le t_r \quad (14)$$

In this period the inventory level in the RW decreases due to the combined effect of the demand (due to the stock displayed in the own warehouse) and the deterioration.

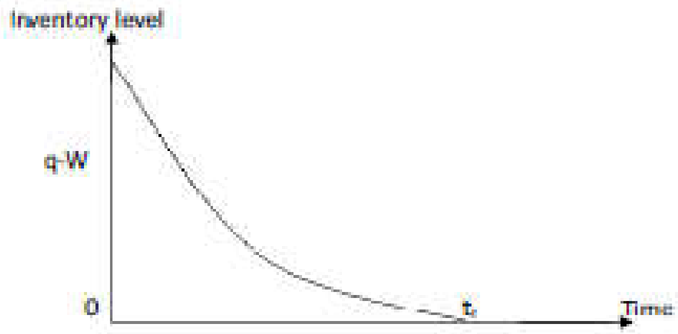

Fig. 2. Retailer's inventory level for rented warehouse

Thus the differential equation governing the model is as follows and depicted in the Fig. 2.

$$\frac{dQ_r(t)}{dt} + \gamma\,\delta\, t^{\delta-1}\, Q_r(t) = -\alpha [Q_0(t)]^{\beta}\ ,\ 0 \le t \le t_r \tag{15}$$

With the boundary condition $Q_r(0)$ =(q-W) and $Q_r(t_r)$ =0, the inventory at any time t in the RW is given by

$$Q_r(t) = \left\{ q - W - \alpha W^{\beta}\left(t_r - \frac{\gamma\, t_r^{\delta+1}}{\delta+1} \right) \right\} e^{-\gamma t^{\delta}}\ ,\ 0 \le t \le t_r \tag{16}$$

At time, $t=t_r$, we have

$$q = W + \alpha W^{\beta}\left(t_r + \frac{\gamma\, t_r^{\delta+1}}{\delta+1} \right) \tag{17}$$

Model formulation for the time period, $t_r \le t \le T$

In this time period, inventory level in the Own warehouse decreases due to the combined effect of the demand and deterioration

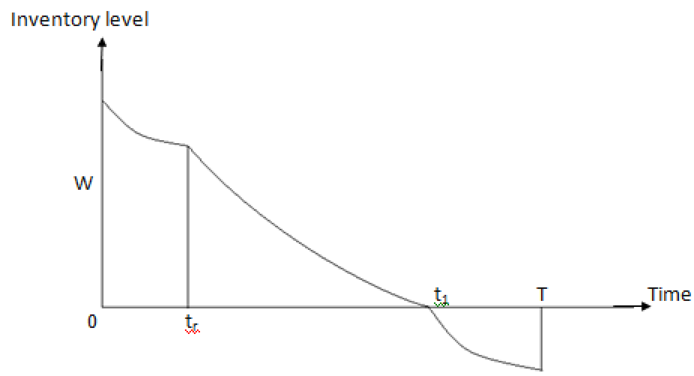

Fig. 3. Retailer's inventory level for Own warehouse

Therefore, the differential equation governing the model is as follows

$$\frac{dQ_0(t)}{dt} + \theta\, t\, Q_0(t) = -\alpha [Q_0(t)]^{\beta}\ ,\qquad t_r \le t \le t_1 \tag{18}$$

and $$\frac{dQ_0(t)}{dt} = -\alpha [Q_0(t)]^{\beta}\ ,\qquad t_1 \le t \le T \tag{19}$$

With the boundary conditions $Q_0(t_1)=0$ and $Q_0(T)=-S$, the solution of the equations (18) and (19), we have

$$Q_0(t)=Q_0(t_r)\, e^{\theta(t_r-t)} - \frac{\alpha}{\theta} W^{\beta}\left\{1 - e^{\theta(t_r-t)}\right\},\ t_r \le t \le t_1 \tag{20}$$

and $Q_0(t) = \alpha W^{\beta}(t_1 - t)\ ,\ t_1 \le t \le T$ (21)

Now, the different costs corresponding to the two-warehouse model which we consider here are follows as:

Holding cost of the items in RW

The holding cost consists for RW during the interval $(0,t_r)$ and is given by

$$HC_r = C_{hr} \int_0^{t_r} Q_r(t)\, dt$$

$$= C_{hr} \int_0^{t_r} \left\{ q - W - W^{\beta}\left(t + \frac{\gamma t^{\delta+1}}{\delta+1}\right)\right\} e^{-\gamma t^{\delta}}\, dt$$

$$= C_h\, \alpha W^{\beta}\left\{ \frac{1}{2}t_r^2 - \left(\frac{\gamma}{\delta+1}\right)^2 \left(t_r^{\delta+1} - \frac{1}{2}t_r^{2\delta+2}\right) + \frac{\gamma\delta}{(\delta+1)(\delta+2)} t_r^{\delta+2}\right\} \tag{22}$$

Holding cost of the items in OW

The holding cost consists for OW during the interval $(0, t_1)$ and is given by

$$HC_0 = C_h \int_0^{t_1} Q_0(t)\, dt$$

$$= C_h \int_0^{t_r} Q_0(t)\, dt + C_h \int_{t_r}^{t_1} Q_0(t)\, dt$$

$$= C_h \int_0^{t_r} W e^{-\theta t} dt + C_h \int_{t_r}^{t_1} \left\{ Q_0(t_r)\, e^{\theta(t_r-t)} - \frac{\alpha}{\theta} W^{\beta}\left(1 - e^{\theta(t_r-t)}\right)\right\} dt$$

$$= \frac{C_h}{\theta}\left\{ W\left(1 - e^{-\theta t_r}\right) + \left(Q_0(t_r) - \alpha W^{\beta}\right)\left(1 - e^{\theta(t_r-t_1)}\right) - \alpha W^{\beta}(t_1 - t_r)\right\} \tag{23}$$

Deterioration Cost of the items in RW

The deterioration cost of the model consists during the interval $(0,t_r)$ and is given by

$$DC_r = C_d \int_0^{t_r} \gamma\delta t^{\delta-1} . Q_r(t)\, dt$$

$$= C_d \int_0^{t_r} \gamma\delta t^{\delta-1} \left\{ q - W - W^{\beta}\left(t + \frac{\gamma t^{\delta+1}}{\delta+1}\right)\right\} e^{-\gamma t^{\delta}}\, dt$$

$$= C_d \gamma\delta \left[\begin{array}{l} \dfrac{(q-W)}{(\delta+2)} t_r^{\delta} - \dfrac{\gamma(q-W)}{(2\delta+2)} t_r^{2\delta+2} \\ \quad - \alpha W^{\beta}\left\{ \dfrac{t_r^{\delta+3}}{(\delta+3)} - \dfrac{t_r^{2\delta+3}}{(2\delta+3)} + \dfrac{\gamma t_r^{2\delta+3}}{(\delta+1)(2\delta+3)} - \dfrac{\gamma t_r^{3\delta+3}}{(\delta+1)(3\delta+3)} \right\} \end{array} \right] \tag{24}$$

Deterioration Cost of the items in OW

The deterioration cost of the model consists during the interval $(0,t_1)$ and is given by

$$DC_0 = C_d \int_0^{t_1} \theta Q_0(t)\, dt$$

$$= C_d \int_0^{t_r} \theta Q_0(t)\, dt + C_d \int_{t_r}^{t_1} \theta Q_0(t)\, dt$$

$$= C_d \int_0^{t_r} \theta W e^{-\theta t} dt + C_d \int_{t_r}^{t_1} \theta \left\{ Q_0(t_r)\, e^{\theta(t_r - t)} - \frac{\alpha}{\theta} W^{\beta} \left(1 - e^{\theta(t_r - t)}\right) \right\} dt$$

$$= C_d \left\{ W\left(1 - e^{-\theta t_r}\right) + \left(Q_0(t_r) - \alpha W^{\beta}\right)\left(1 - e^{\theta(t_r - t_1)}\right) - \alpha W^{\beta}\left(t_1 - t_r\right) \right\} \quad (25)$$

Shortages Cost: The shortages cost consists during the time (t_1, T) is given by

$$SC = C_s \int_{t_1}^{T} Q_0(t)\, dt$$

$$= C_s\, \alpha W^{\beta} \int_{t_1}^{T} \left(t_1 - t\right) dt$$

$$= C_s\, \alpha W^{\beta} \left\{ T t_1 - \frac{1}{2}\left(T^2 + t_1^2\right) \right\} \quad (26)$$

Purchasing cost:PC = c q (27)

Fixed cost: F_1 (28)

Total cost of Single warehouse system

Total cost govern by above all the costs

$TC_2(t_r)$ = Holding cost (in both RW and OW) + Cost of Defective Items (in both RW and OW) + Shortage cost + Purchasing Cost+Fixed cost.

$$= C_h\, \alpha W^{\beta} \left\{ \frac{1}{2} t_r^2 - \left(\frac{\gamma}{\delta + 1}\right)^2 \left(t_r^{\delta+1} - \frac{1}{2} t_r^{2\delta+2} \right) + \frac{\gamma\delta}{(\delta+1)(\delta+2)} t_r^{\delta+2} \right\}$$

$$+ \frac{C_h}{\theta} \left\{ W\left(1 - e^{-\theta t_r}\right) + \left(Q_0(t_r) - \alpha W^{\beta}\right)\left(1 - e^{\theta(t_r - t_1)}\right) - \alpha W^{\beta}\left(t_1 - t_r\right) \right\}$$

$$+ C_d \gamma\delta \left[\frac{(q - W)}{(\delta + 2)} t_r^{\delta} - \frac{\gamma(q - W)}{(2\delta + 2)} t_r^{2\delta+2} - \alpha W^{\beta} \left\{ \frac{t_r^{\delta+3}}{(\delta+3)} - \frac{t_r^{2\delta+3}}{(2\delta+3)} + \frac{\gamma\, t_r^{2\delta+3}}{(\delta+1)(2\delta+3)} - \frac{\gamma\, t_r^{3\delta+3}}{(\delta+1)(3\delta+3)} \right\} \right]$$

$$+ C_d \left\{ W\left(1 - e^{-\theta t_r}\right) + \left(Q_0(t_r) - \alpha W^{\beta}\right)\left(1 - e^{\theta(t_r - t_1)}\right) - \alpha W^{\beta}\left(t_1 - t_r\right) \right\}$$

$$+ C_s\, \alpha W^{\beta} \left\{ T t_1 - \frac{1}{2}\left(T^2 + t_1^2\right) \right\} + c\, q + F_1 \quad (29)$$

From eqns. (17) (29), it is well known that the average total cost function depends only on the continuous variable, t_r. Now our object is to determine the optimal value of t_r in order to keep $TC_2(t_r)$ minimum.

From the necessary condition for the minimization of $TC_2(t_r)$, we have

$$\frac{d\{TC_2(t_r)\}}{dt_r} = 0 \tag{30}$$

And the necessary condition for the minimization of $TC_2(t_r)$, is

$$\frac{d\{TC_2(t_r)\}}{dt_r} > 0 \tag{31}$$

To determine the optimal values of the objective function, we determine the following algorithm.

Step 1: Input the values of all parameters.
Step 2: If M≥0 go to step 3; otherwise go to step 5;
Step 3: Solve equation (30) for the optimal value of T^*, satisfying the equation (31).
Step 4: Compute $TC_2(T^*)$; q^* and T^*; and output $TC_2(T^*)$; q^* and T^*; stop:
Step 5: If M < 0, solve equation (12) for optimal value of q^*.
Step 6: Compute $TC_1(q^*)$; $q^*$and T^*; and output $TC_1(q^*)$; q^* and T^*; stop:

Numerical Example

The model developed above is illustrated by the following numerical example.

Example; Let c=15, α=30, β=0.16, C_h=12, C_{hr}=15, C_d=4, C_{dr}=5, C_s=10, ϒ=0.01, δ=2, W=400, θ=0.9, F_1=150 in appropriate units. Then using lingo software we have I_0=350, Q=411.6098, t_r=0.1483682, t_1=2.165876, HC=1595.727, HC_r=12.91886, DC=478.7182, DC_r=0.127995, SC=488.2813

And optimum total cost is TC=8411.512

Sensitivity Analysis

Table 1. Optimal replenishment policy for different values of demand parameters

α	β in %	q	TC	M	Use of RW
	-20	403.195	8039.276	+	Yes
	-10	403.516	6723.503	+	Yes
10	0	403.869	6693.983	+	Yes
	+10	404.259	6661.449	+	Yes
	+20	404.687	9846.404	+	Yes
	-20	406.389	6479.910	+	Yes
	-10	407.032	6422.575	+	Yes
20	0	407.739	6357.305	+	Yes
	+10	408.518	9430.357	+	Yes
	+20	409.376	6387.260	+	Yes
	-20	409.584	6800.552	+	Yes
	-10	410.548	6637.362	+	Yes
30	0	411.609	8411.512	+	Yes
	+10	412.778	7802.410	+	Yes
	+20	414.063	6376.601	+	Yes
	-20	412.779	7773.782	+	Yes
	-10	414.065	6376.622	+	Yes
40	0	415.479	6399.422	+	Yes
	+10	417.037	6424.516	+	Yes
	+20	418.751	6452.135	+	Yes
	-20	415.973	6407.383	+	Yes
	-10	417.581	6433.278	+	Yes
50	0	419.349	6461.778	+	Yes
	+10	421.296	6493.145	+	Yes
	+20	423.439	6527.669	+	Yes

Observations

From the Table 1, it is quite clear that the demand parameters more sensitive to the inventory system using the two-warehouse model.

Graphically change of Total cost with several parameters

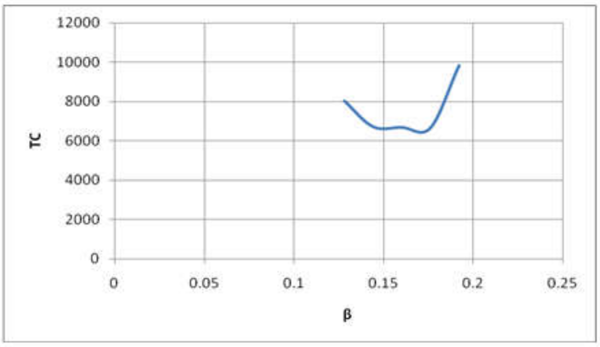

Fig 4.

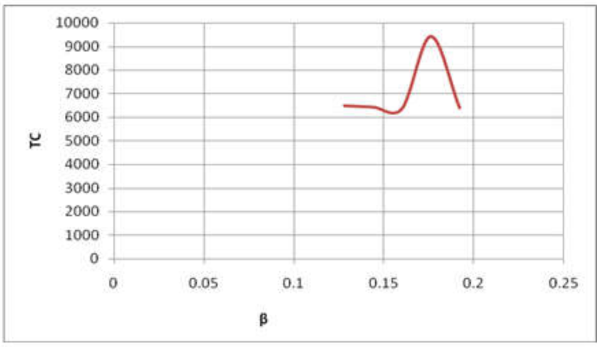

Fig 5.

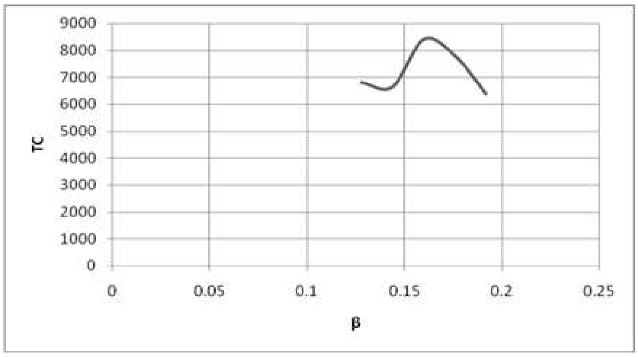

Fig 6.

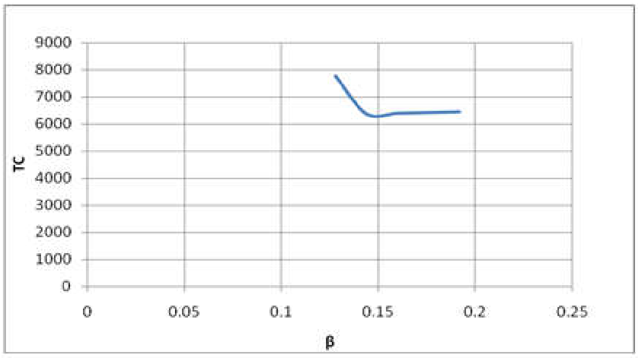

Fig 7.

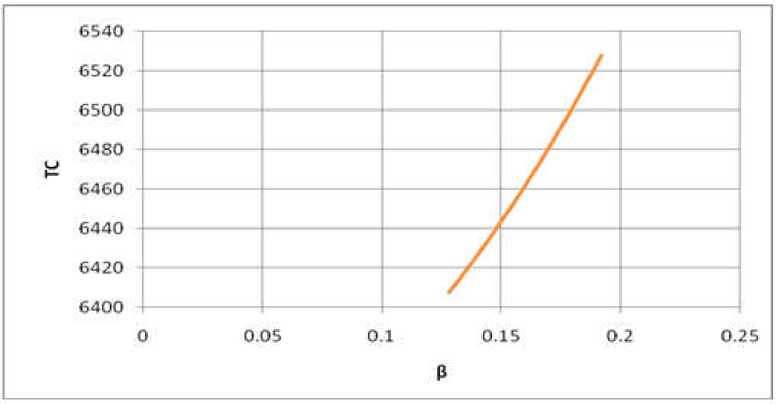

Fig 8.

Figure 4 to 8 shows that the change of Total cost with β corresponding to α=10, 20, 30, 40, 50 respectively

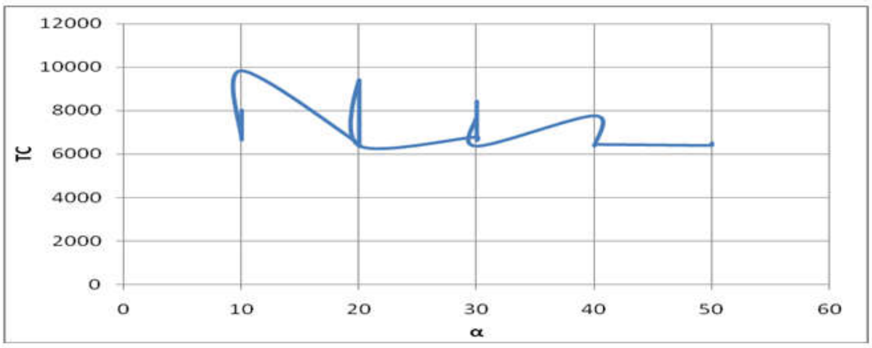

Figure 9. Shows that the change of Total cost with α

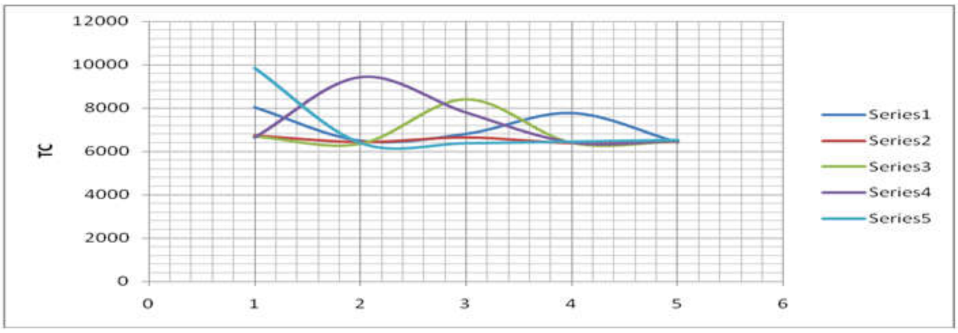

Figure 10.

Sensitivity

To study the effect of other parameters on the optimal policy a sensitivity analysis was done, considering previous example with demand parameters α=30 and β=0.16.

Table 2. Variation of the optimal values with respect to different parameters

Parameters	% Change	t_r	t_1	q	TC	M	Use of RW
	-10	0.313	3.161	362.408	6036.432	+	Yes
	-5	0.914	1.372	387.091	7988.947	+	Yes
W	0	0.148	2.166	411.609	8411.512	+	Yes
	+5	0.202	2.640	435.977	8771.173	+	Yes
	+10	0.254	1.907	460.205	10005.380	+	Yes
	-20	0.148	0.839	411.609	7212.532	+	Yes
	-10	0.148	0.839	411.609	7829.947	+	Yes
c	0	0.148	2.166	411.609	8411.512	+	Yes
	+10	0.148	1.963	411.609	9150.613	+	Yes
	+20	0.148	0.839	411.609	9682.191	+	Yes
	-20	0.148	2.166	411.609	8411.506	+	Yes
	-10	0.148	2.166	411.609	8411.506	+	Yes
ϒ	0	0.148	2.166	411.609	8411.512	+	Yes
	+10	0.148	2.166	411.609	8411.515	+	Yes
	+20	0.148	2.166	411.609	8411.518	+	Yes
	-5	0.148	2.166	411.610	8411.517	+	Yes
δ	0	0.148	2.166	411.609	8411.512	+	Yes
	+5	0.148	2.166	411.609	8411.508	+	Yes
	-20	0.185	2.047	414.513	8556.843	+	Yes
	-10	0.166	2.317	412.900	8350.193	+	Yes
θ	0	0.148	2.166	411.609	8411.512	+	Yes
	+10	0.135	1.835	410.554	8547.677	+	Yes
	+20	0.123	4.166	409.674	8334.266	+	Yes

Graphically Variation of the optimal values with respect to different parameters

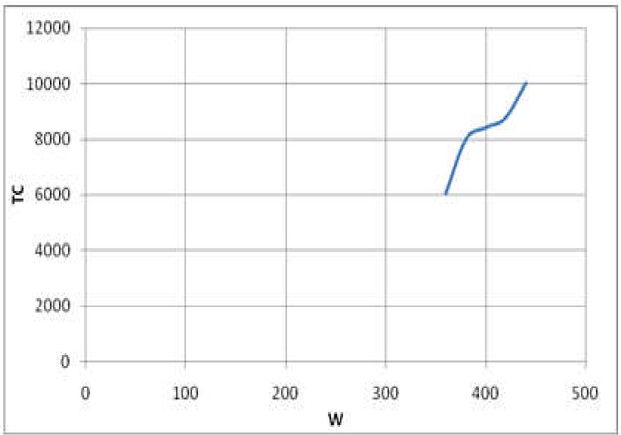

Fig 11.

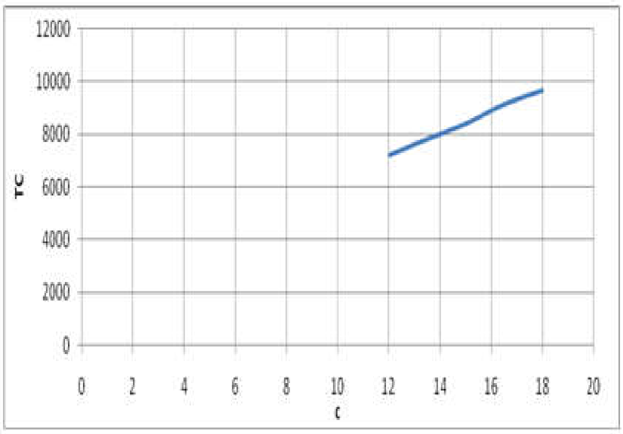

Fig 12.

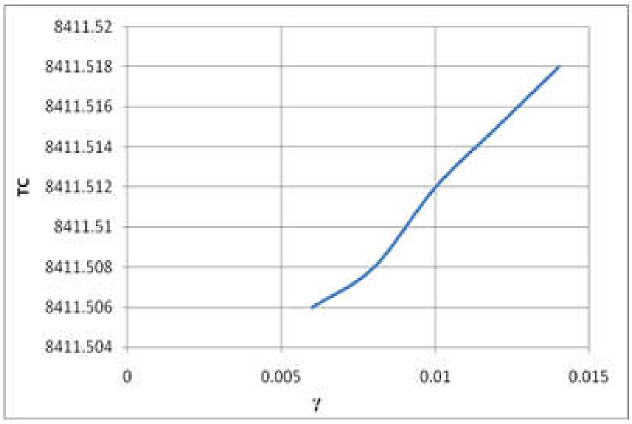

Fig 13.

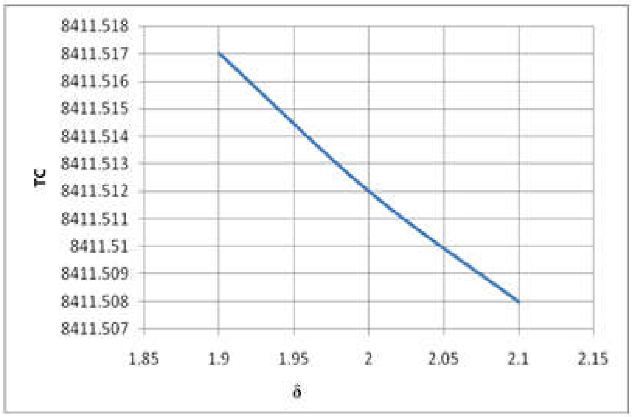

Fig 14.

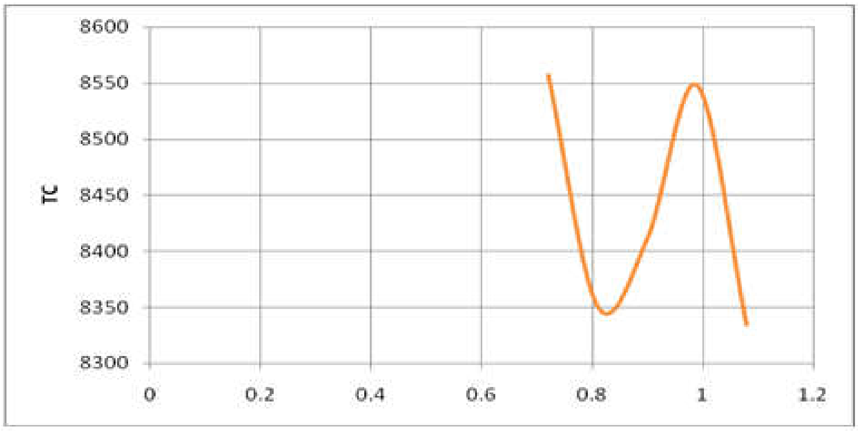

Fig 15

Figures 11 to 15 shows that change of Total Cost with the parameters w, c, γ, δ, θ respectively

Observations

From Table 2, it is observed that

- More the capacity of the OW more costly to the two-warehouse inventory system. If the retailer will purchase more inventories, then consequently the cost increases.
- The total cost increases as the purchasing cost increases. It may be understood, decrease in the purchasing cost also encourages the retailer to store more inventory, as a consequences the profit increases.
- From the graph and sensitivity analysis it is observed that the optimal policy is more sensitive to parameters w, c and θ as compared to other parameters.
- The effect of the parameters γ and δ on the optimal policy is very slightly significant.

Table 3. Variation of the optimal values with respect to different cost parameters

Holding cost	Change in %	q	TC	M	Use of RW
	-20	411.609	6939.163	+	Yes
	-10	411.609	6395.281	+	Yes
C_h	0	411.609	8411.512	+	Yes
	+10	411.609	8869.008	+	Yes
	+20	411.609	9046.451	+	Yes
	-20	411.609	8404.928	+	Yes
	-10	411.609	8410.220	+	Yes
C_{hr}	0	411.609	8411.512	+	Yes
	+10	411.609	8412.804	+	Yes
	+20	411.609	8414.096	+	Yes

Graphically Variation of optimum cost with holding cost:

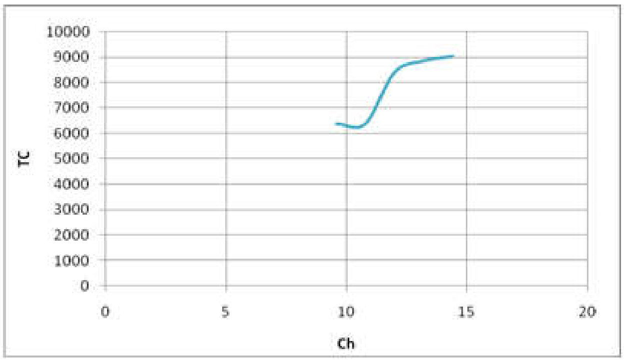

Fig 16.

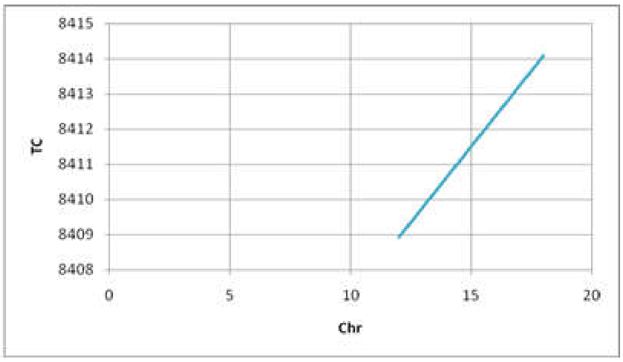

Fig 17

Observations

From the graph and sensitivity analysis it is observed that the optimal policy is more sensitive to parameter C_h and low sensitive to C_{hr}.

Table 4. Variation of the optimal values with respect to cost parameters

Holding cost	Change in %	q	TC	M	Use of RW
	-20	411.609	8412.586	+	Yes
	-10	411.609	6399.414	+	Yes
C_d	0	411.609	8411.512	+	Yes
	+10	411.609	8733.171	+	Yes
	+20	411.609	8794.680	+	Yes
	-40	411.609	6537.560	+	Yes
	-20	411.609	6545.629	+	Yes
C_s	0	411.609	8411.512	+	Yes
	+20	411.609	6538.939	+	Yes
	+40	411.609	6536.338	+	Yes

Graphically Variation of optimum cost with deterioration and shortage cost

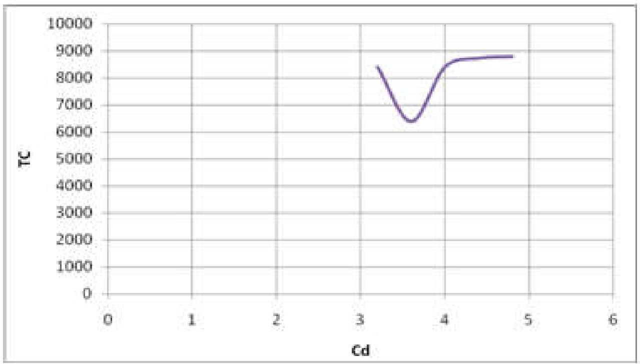

Fig 18.

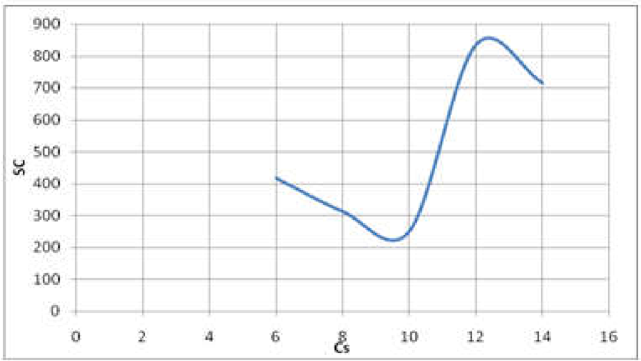

Fig 19.

Observations

From the graph and sensitivity analysis it is observed that the optimal policy is more sensitive to parameters C_d and C_s.

Conclusion

In this paper, inventory model for a single item with inventory-dependent demand rate is developed and extended to the case with two warehouses (OW and RW) to determine whether the retailer should hire a RW or not. The demand rate is assumed to be a polynomial form of initial inventory level in OW. It is considered that the deterioration rate per unit items in the RW and OW is different due to different preservation environments as a consequence the holding costs per unit item in RW and OW are also different and consider shortages. Another feature here is that a sufficient and necessary condition is provided for the decision-maker to determine whether to use the rented warehouse. Another direction of future development of research is that, in this paper we have assumed that the holding cost per unit item in the RW is greater than that in OW, in further studies the opposite situation and related implications may also be discussed and one can extend this model by incorporating some more practical situations, such as inflation, incremental quantity discount a, multiple items and fuzzy demand etc.

REFERENCES

Balakrishnan, A., Pangburn, M.S., Stavrulaki, E. 2004. Stack them high, let 'em fly": Lot-sizing policies when inventories stimulate demand. *Manag. Sci*., 50(5), 630–644.

Balkhi, T.Z., Benkherouf, L. 2004. On an inventory model for deteriorating items with stock dependent and time varying demand rates. *Comput. Oper. Res.* 31, 223–240.

Bhunia, A.K., Maiti, M. 1998. A two-warehouse inventory model for deteriorating items with a linear trend in demand and shortages. J. Oper. Res. Soc. 49, 287–292.

Chang, J., Dye, C.Y. 1999. An EOQ model for deteriorating items with time varying demand and partial backlogging. *J. Oper. Res. Soc*., 50, 1176–1182.

Datta, T.K., Pal, A.K. 1990. A note on an inventory model with inventory-level-dependent demand rate. *J. Oper. Res. Soc.*, 41(10), 971–975.

Datta, T.K., Paul, K., Pal, A.K. 1998. Demand promotion by up-gradation under stock-dependent demand situation—a model. (Int. *J. Prod. Econ*., 55, 31–38 .

Dave, U. 1988. On the EOQ models with two levels of storage. Opsearch 25, 190–196.

Dave, U., Patel, L.K. 1981. (T, Si) policy inventory model for deteriorating items with time proportional demand. *J. Oper. Res. Soc.*, 32, 137–142.

Deb, M., Chaudhuri, K.S. 1986. An EOQ model for items with finite rate of production and variable rate of deterioration. Opsearch 23, 175–181.

Dye, C.Y., Ouyang, L.Y., Hsieh, T.P. 2007. Deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate. Eur. J. Oper. Res. 178(3), 789–807.

Ghare, P.M., Schrader, G.H. 1963. A model for exponentially decaying inventory system. (Int. J. Prod. Econ. 21, 449–460.

Ghosh, S.K., Khanra, S., Chaudhuri, K.S. 2011. An inventory model for a deteriorating item with two levels of storage and stock-dependent demand. *Int. J. Math. Oper. Res*., 3(2), 186 197

Hartley, V.R.1976. Operations research—a managerial emphasis, pp. 315–317. Good Year, Santa Monica (1976).

Hsieh, T.P., Dye, C.Y., Ouyang, and L.Y. 2008. Determining optimal lot size for a two-warehouse system with deterioration and shortages using net present value. Eur. J. Oper. Res. 191, 182–192.

Kar, S., Bhunia, A.K., Maiti, M. 2001. Deterministic inventory model with two levels of storage, a linear trend in demand and a fixed time horizon. Comput. Oper. Res. 28, 1315–1331.

Pakkala, T.P.M., Achary, K.K. 1992. A deterministic inventory model for deteriorating items with two warehouses and finite replenishment rate. Eur. J. Oper. Res. 57, 71–76.

Roy, A., Maiti, M.K., Kar, S., Maiti, M. 2009. An inventory model for a deteriorating item with displayed stock dependent demand under fuzzy inflation and time discounting over a random planning horizon. *Appl. Math. Model*., 33(2), 744–759.

Sajadieh, M.S., Thorstenson, A., Jokar, M.R.A. 2010. An integrated vendor-buyer model with stock dependent demand. Transport. Res. E Logist.Transport. Rev. 46(6), 963–974.

Sharma, K.V.S. 1987. A deterministic order-level inventory model for deteriorating items with two storage facilities. *Eur. J. Oper.* Res. 29, 70–72.

Singh, S.R., Singh, C. 2010. Two echelon supply chain model with imperfect production, for Weibull distribution deteriorating items under imprecise and inflationary environment. *Int. J. Oper. Res. Optima*. 1(1), 9–25.

Singh, S.R., Singh, C., Singh, T.J. 2007. Optimal policy for decaying items with stock-dependent demand under inflation in a supply chain. Int. Rev Pure Appl. Math 3(2), 189–197.

Teng, J.T., Chang, and C.T. 2005. Economic production quantity models for deteriorating items with price and stock dependent demand. *Comput. Oper. Res*., 32, 297–308.

Teng, J.T., Ouyang, L.Y., Cheng, M.C. 2005. An EOQ model for deteriorating items with power-form stock-dependent demand. Inform. *Manag. Sci.,* 16(1), 1–16.

Vishnoi, M., Shon, S.K.2010. Two levels of storage model for non-instantaneous deteriorating items with stock dependent demand, time varying partial backlogging under permissible delay in payment. *Int. J. Oper. Res. Optima*. 1(1), 133–147.

Wu, K.S., Ouyang, L.Y., Yang, C.T. 2006. An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. *Int. J. Prod. Econ*., 101, 369

Yang, H.L. 2006. Two-warehouse partial backlogging inventory models for deteriorating items under inflation. Int. J. Prod. Econ. 103, 362–370.

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