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Research Article

ON A NEW DYNAMICS ON KAHLER MANIFOLD USING LOCAL CANONICAL BASIS

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ABSTRACT

In his paper we obtained a canonical local basis $\{J_i\}, i = \overline{1,6}$ of vector bundle V on Clifford $K\ddot{a}hler$ manifold (M, V).the paths of semispray on Clifford $K\ddot{a}hler$ manifold are infact the solutions of Euler-Lagrange equations.

INTRODUCTION

It is well-known that modern differential geometry express explicitly the dynamics of Lagrangians.

Therefore we explain that if M is an m-dimensional configuration manifold and $L : TM \rightarrow R$ is a regular lagrangian function, then there is a unique vector field ξ on TM such that dynamics equations is determined by:

$$i_\xi \Phi_l = d\mathrm{E}_l \qquad \rightarrow \qquad (1)$$

Where Φ_l indicates the symplectic form and E_l is the energy associated to L [De Leon, 1989; Tekkoyun, 2005].

The Triple (TM, Φ_l, ξ) in named lagrangian system on the tangent bundle TM.

It is known , there are many studies about Lagrangian mechanics , formalisms , systems and equations such as that real , complex , paracomplex and other analogues [De Leon, 1989; Tekkoyun, 2005] and there in. so , it may be possible to produced different analogues in different spaces. The goal of finding new dynamics equations is both a new expansion and contribution to science to explain physical events. Sir William Rowan Hamilton invented quaternions as an extension to the complex numbers. Hamilton's defining relation is most succinctly written as:

$$i^2 = j^2 = k^2 = ijk = -1 \qquad \rightarrow \qquad (2)$$

If it is compared to the calculus of vectors, quaternion's have slipped into the realm of obscurity. They do however still find use in the in the computation of rotations. A lot of physical laws in classical, relativistic, and quantum mechanics can be written pleasantly by means of quaternions. Some physicists hope they will find deeper understanding of the universe by restating basic principles in terms of quaternion algebra. It is well-known that quaternions are useful for representing rotations in both quantum and classical mechanics [http://www.stahlke.org/dan/phys-papers/quaternion-paper.pdf]. Clifford manifolds are also quaternion manifolds [Tekkoyun, ?].

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2-preliminaries

In this paper, all mathematical objects and mappings are assumed to be smooth, i.e. infinitely differentiable and Einstein convention of summarizing is adopted. $\mathcal{F}(M), \mathcal{X}(M) and\ \Lambda^1(M)$ define the set of functions on M, the set of vector fields on M and the set of 1-forms on M,respectively.

2.1 Theorem

Let f be differentiable ϕ ,ψ are 1-form, then [Abdulla Eid, 2008]:

- $d(f\phi) = df\wedge\phi + fd\phi$
- $d(\phi\wedge\psi)$=d$\phi\wedge\psi - \phi\wedge d\psi$

2.2 Definition (Kronecker's delta)

Kronecker's delta denote by δ and defined as follows [7,8]:

$$\delta_i^j = \begin{cases} 1 \ ; & if \quad i = j \\ 0 \ ; & if \quad i \neq j \end{cases}$$

2.3 Clifford $Kähler$ Manifolds

Now, here we extend and rewrite the main concepts and structures given in [5,9,10] Let M be a real smooth manifold of dimension m. Suppose that there is a 6-dimensional vector bundle V consisting of $F_i(i = 1,2, \ldots ,6)$ tensors of type (1,1) over M. Such a local basis $\{F_1, F_2, \ldots, F_6\}$ is named a canonical local basis of the bundle V in a neighborhood U of M. Then V is called an almost Clifford structure in M. The pair $(M, V)$is named an almost Clifford manifold with V. Thus, an almost Clifford manifold M is of $dimension\ m = 8n$. If there exists on (M, V) a global basis $\{F_1, F_2, \ldots, F_6\}$, then (M, V) is called an almost Clifford manifold; the basis $\{F_1, F_2, \ldots, F_6\}$ is said to be a global basis for V.

An almost Clifford connection on the almost Clifford manifold (M, V) is a linear connection $\nabla\ on\ M$ which preserves by parallel transport the vector bundle V. This means that if Φ is a cross-section (local-global) of the bundle V. Then $\nabla_X\Phi$ is also a cross-section (local-global, respectively) of V , X being an arbitrary vector field of M.

If for any canonical basis $\{J_i\}$, $i = \overline{1,6}\ of\ V$ in a coordinate neighborhood U, the identities

$$g(J_iX, J_iY) = g(X, Y)\ ,\ \forall X, Y \in \chi(M), i = 1,2, \ldots ,6 \quad \rightarrow \qquad (3)$$

Hold, the triple (M, g, V) is called an almost Clifford Hermitian manifold or metric Clifford manifold denoting by V an almost Clifford structure V and by g a Riemannian metric and by (g, V) an almost Clifford metric structure.

Since each $J_i(i = 1,2, \ldots ,6)$ is almost Hermitian structure with respect to g, setting

$$\Phi_i(X, Y) = g(J_iX, Y), \quad i = 1,2, \ldots ,6 \qquad \rightarrow \qquad (4)$$

For any vector fields $X\ and\ Y$, we see that Φ_i are 6-local 2-forms.

If the Levi-Civita connection ∇= $\nabla^g\ on\ (M, g, V)$ preserves the vector bundle V by parallel transport, then (M, g, V) is named a Clifford $Kähler$ manifold, and an almost Clifford structure Φ_i of M is said to be a Clifford $Kähler$ structure. Suppose that let

$$\{x_i\ , x_{n+i}\ , x_{2n+i}\ , x_{3n+i}\ , x_{4n+i}\ , x_{5n+i}\ , x_{6n+i}\ , x_{7n+i}\}\ , i = \overline{1, n}$$

be a real coordinate system on (M, V). Then we denote by

$$\left\{\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_{n+i}}, \frac{\partial}{\partial x_{2n+i}}, \frac{\partial}{\partial x_{3n+i}}, \frac{\partial}{\partial x_{4n+i}}, \frac{\partial}{\partial x_{5n+i}}, \frac{\partial}{\partial x_{6n+i}}, \frac{\partial}{\partial x_{7n+i}}\right\},$$
$$\{dx_i, dx_{n+i}, dx_{2n+i}, dx_{3n+i}, dx_{4n+i}, dx_{5n+i}, dx_{6n+i}, dx_{7n+i}\}$$

The natural bases over R of the tangent space $T(M)$ and the cotangent space $T^*(M)\ of\ M$, respectively.
By structure $\{J_1\ ,J_2\ ,J_3\ ,J_4\ ,J_5\ ,J_6\}$ the following expressions are given

$$J_1\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{n+i}} \qquad J_2\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{2n+i}} \qquad J_3\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial x_{3n+i}}$$

$$J_1\left(\frac{\partial}{\partial x_{n+i}}\right)=-\frac{\partial}{\partial x_i}\quad J_2\left(\frac{\partial}{\partial x_{n+i}}\right)=-\frac{\partial}{\partial x_{4n+i}}\quad J_3\left(\frac{\partial}{\partial x_{n+i}}\right)=-\frac{\partial}{\partial x_{5n+i}}$$
$$J_1\left(\frac{\partial}{\partial x_{2n+i}}\right)=\frac{\partial}{\partial x_{4n+i}}\quad J_2\left(\frac{\partial}{\partial x_{2n+i}}\right)=-\frac{\partial}{\partial x_i}\quad J_3\left(\frac{\partial}{\partial x_{2n+i}}\right)=-\frac{\partial}{\partial x_{6n+i}}$$
$$J_1\left(\frac{\partial}{\partial x_{3n+i}}\right)=\frac{\partial}{\partial x_{5n+i}}\quad J_2\left(\frac{\partial}{\partial x_{3n+i}}\right)=\frac{\partial}{\partial x_{6n+i}}\quad J_3\left(\frac{\partial}{\partial x_{3n+i}}\right)=-\frac{\partial}{\partial x_i}$$
$$J_1\left(\frac{\partial}{\partial x_{4n+i}}\right)=-\frac{\partial}{\partial x_{2n+i}}\quad J_2\left(\frac{\partial}{\partial x_{4n+i}}\right)=\frac{\partial}{\partial x_{n+i}}\quad J_3\left(\frac{\partial}{\partial x_{4n+i}}\right)=\frac{\partial}{\partial x_{7n+i}}$$
$$J_1\left(\frac{\partial}{\partial x_{5n+i}}\right)=-\frac{\partial}{\partial x_{3n+i}}\quad J_2\left(\frac{\partial}{\partial x_{5n+i}}\right)=-\frac{\partial}{\partial x_{7n+i}}\quad J_3\left(\frac{\partial}{\partial x_{5n+i}}\right)=\frac{\partial}{\partial x_{n+i}}$$
$$J_1\left(\frac{\partial}{\partial x_{6n+i}}\right)=\frac{\partial}{\partial x_{7n+i}}\quad J_2\left(\frac{\partial}{\partial x_{6n+i}}\right)=-\frac{\partial}{\partial x_{3n+i}}\quad J_3\left(\frac{\partial}{\partial x_{6n+i}}\right)=\frac{\partial}{\partial x_{2n+i}}$$
$$J_1\left(\frac{\partial}{\partial x_{7n+i}}\right)=-\frac{\partial}{\partial x_{6n+i}}\quad J_2\left(\frac{\partial}{\partial x_{7n+i}}\right)=\frac{\partial}{\partial x_{5n+i}}\quad J_3\left(\frac{\partial}{\partial x_{7n+i}}\right)=-\frac{\partial}{\partial x_{4n+i}}$$
$$J_4\left(\frac{\partial}{\partial x_i}\right)=\frac{\partial}{\partial x_{4n+i}}\quad J_5\left(\frac{\partial}{\partial x_i}\right)=\frac{\partial}{\partial x_{5n+i}}\quad J_6\left(\frac{\partial}{\partial x_i}\right)=\frac{\partial}{\partial x_{6n+i}}$$
$$J_4\left(\frac{\partial}{\partial x_{n+i}}\right)=-\frac{\partial}{\partial x_{2n+i}}\quad J_5\left(\frac{\partial}{\partial x_{n+i}}\right)=-\frac{\partial}{\partial x_{3n+i}}\quad J_6\left(\frac{\partial}{\partial x_{n+i}}\right)=-\frac{\partial}{\partial x_{7n+i}}$$
$$J_4\left(\frac{\partial}{\partial x_{2n+i}}\right)=\frac{\partial}{\partial x_{n+i}}\quad J_5\left(\frac{\partial}{\partial x_{2n+i}}\right)=-\frac{\partial}{\partial x_{7n+i}}\quad J_6\left(\frac{\partial}{\partial x_{2n+i}}\right)=-\frac{\partial}{\partial x_{3n+i}}$$
$$J_4\left(\frac{\partial}{\partial x_{3n+i}}\right)=-\frac{\partial}{\partial x_{7n+i}}\quad J_5\left(\frac{\partial}{\partial x_{3n+i}}\right)=\frac{\partial}{\partial x_{n+i}}\quad J_6\left(\frac{\partial}{\partial x_{3n+i}}\right)=\frac{\partial}{\partial x_{2n+i}}$$
$$J_4\left(\frac{\partial}{\partial x_{4n+i}}\right)=-\frac{\partial}{\partial x_i}\quad J_5\left(\frac{\partial}{\partial x_{4n+i}}\right)=\frac{\partial}{\partial x_{6n+i}}\quad J_6\left(\frac{\partial}{\partial x_{4n+i}}\right)=\frac{\partial}{\partial x_{5n+i}}$$
$$J_4\left(\frac{\partial}{\partial x_{5n+i}}\right)=\frac{\partial}{\partial x_{6n+i}}\quad J_5\left(\frac{\partial}{\partial x_{5n+i}}\right)=-\frac{\partial}{\partial x_i}\quad J_6\left(\frac{\partial}{\partial x_{5n+i}}\right)=-\frac{\partial}{\partial x_{4n+i}}$$
$$J_4\left(\frac{\partial}{\partial x_{6n+i}}\right)=-\frac{\partial}{\partial x_{5n+i}}\quad J_5\left(\frac{\partial}{\partial x_{6n+i}}\right)=-\frac{\partial}{\partial x_{4n+i}}\quad J_6\left(\frac{\partial}{\partial x_{6n+i}}\right)=-\frac{\partial}{\partial x_i}$$
$$J_4\left(\frac{\partial}{\partial x_{7n+i}}\right)=\frac{\partial}{\partial x_{3n+i}}\quad J_5\left(\frac{\partial}{\partial x_{7n+i}}\right)=\frac{\partial}{\partial x_{2n+i}}\quad J_6\left(\frac{\partial}{\partial x_{7n+i}}\right)=\frac{\partial}{\partial x_{n+i}}$$

3- Lagrangian Mechanics

In this section, we introduce Euler-Lagrange equations for quantum and classical mechanics by means of canonical local basis $\{J_i\}, i=\overline{1,6}\ of\ V$on Clifford $K\ddot{a}hler$ manifold (M,V). We say that the Euler-Lagrange equations using basis $\{J_1,J_2,J_3\}$ of V on (R^{8n},V) are introduce in [Tekkoyun, ?]. In this study, we obtain that they are the same as the obtained by operators $J_1, J_2, J_3\ of\ V$ on Clifford $K\ddot{a}hler$ manifold (M,V).

If we express them, they are respectively:

First:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right)+\frac{\partial L}{\partial x_{n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right)-\frac{\partial L}{\partial x_i}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right)+\frac{\partial L}{\partial x_{4n+i}}=0,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right)+\frac{\partial L}{\partial x_{5n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right)-\frac{\partial L}{\partial x_{2n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right)-\frac{\partial L}{\partial x_{3n+i}}=0,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right)+\frac{\partial L}{\partial x_{7n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right)-\frac{\partial L}{\partial x_{6n+i}}=0.$$

Second:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right)+\frac{\partial L}{\partial x_{2n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right)-\frac{\partial L}{\partial x_{4n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right)-\frac{\partial L}{\partial x_i}=0,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right)+\frac{\partial L}{\partial x_{6n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right)+\frac{\partial L}{\partial x_{n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right)-\frac{\partial L}{\partial x_{7n+i}}=0,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right)-\frac{\partial L}{\partial x_{3n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right)+\frac{\partial L}{\partial x_{5n+i}}=0.$$

Third:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right)+\frac{\partial L}{\partial x_{3n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right)-\frac{\partial L}{\partial x_{5n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right)-\frac{\partial L}{\partial x_{6n+i}}=0,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right)-\frac{\partial L}{\partial x_i}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right)+\frac{\partial L}{\partial x_{7n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right)+\frac{\partial L}{\partial x_{n+i}}=0,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right)+\frac{\partial L}{\partial x_{2n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right)-\frac{\partial L}{\partial x_{4n+i}}=0.$$

Here, only we derive Euler-Lagrange equations using operators J_4, J_5, J_6 of V on Clifford $K\ddot{a}hler$ manifold (M,V).
Fourth:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right)+\frac{\partial L}{\partial x_{4n+i}}=0,\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right)-\frac{\partial L}{\partial x_{2n+i}}=0,\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right)+\frac{\partial L}{\partial x_{n+i}}=0$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right)-\frac{\partial L}{\partial x_{7n+i}}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right)-\frac{\partial L}{\partial x_i}=0,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right)+\frac{\partial L}{\partial x_{6n+i}}=0,$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right)-\frac{\partial L}{\partial x_{5n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right)+\frac{\partial L}{\partial x_{3n+i}}=0.$$

Such that the equations are named Euler-Lagrange equations structured on Clifford $K\ddot{a}hler$ manifold (M,V) by means of $\Phi_L^{J_4}$ and in the case, the triple $(M,\Phi_L^{J_4},\xi)$ is said to be a mechanical system on Clifford $K\ddot{a}hler$ manifold (M,V).
Fifth:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right)+\frac{\partial L}{\partial x_{5n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right)-\frac{\partial L}{\partial x_{3n+i}}=0\,,\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right)-\frac{\partial L}{\partial x_{7n+i}}=0\,,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right)+\frac{\partial L}{\partial x_{n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right)+\frac{\partial L}{\partial x_{6n+i}}=0\,,\ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right)-\frac{\partial L}{\partial x_i}=0\,,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right)-\frac{\partial L}{\partial x_{4n+i}}=0\,,\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right)+\frac{\partial L}{\partial x_{2n+i}}=0$$

Such that the equations are named Euler-Lagrange equations structured on Clifford $K\ddot{a}hler$ manifold (M,V) by means of $\Phi_L^{J_5}$ and in the case, the triple $(M,\Phi_L^{J_5},\xi)$ is called a mechanical system on Clifford $K\ddot{a}hler$ manifold(M,V).

Sixth, we present Euler-Lagrange equations for quantum and classical mechanics by means of $\Phi_L^{J_6}$ on Clifford $K\ddot{a}hler$ manifold(M,V). Let J_6 be a local basis on Clifford $K\ddot{a}hler$ manifold(M,V).and $\{x_i\,,x_{n+i}\,,x_{2n+i}\,,x_{3n+i}\,,x_{4n+i}\,,x_{5n+i}\,,x_{6n+i}\,,x_{7n+i}\}\,,i=\overline{1\,,n}$ be its coordinate functions.

Let semisparay be the vector field ξ defined by:

$$\xi=X^i\frac{\partial}{\partial x_i}+X^{n+i}\frac{\partial}{\partial x_{n+i}}+X^{2n+i}\frac{\partial}{\partial x_{2n+i}}+X^{3n+i}\frac{\partial}{\partial x_{3n+i}}+X^{4n+i}\frac{\partial}{\partial x_{4n+i}}$$
$$+X^{5n+i}\frac{\partial}{\partial x_{5n+i}}+X^{6n+i}\frac{\partial}{\partial x_{6n+i}}+X^{7n+i}\frac{\partial}{\partial x_{7n+i}}\qquad\rightarrow\qquad(5)$$

Where

$$X^i=\dot{x}_i\,,X^{n+i}=\dot{x}_{n+i}\,,X^{2n+i}=\dot{x}_{2n+i}\,,X^{3n+i}=\dot{x}_{3n+i}\,,X^{4n+i}=\dot{x}_{4n+i}$$
$$X^{5n+i}=\dot{x}_{5n+i}\,,X^{6n+i}=\dot{x}_{6n+i}\,,X^{7n+i}=\dot{x}_{7n+i}.$$

This equation (5) can be written concise manner

$$\xi=\sum_{a=0}^{7}X^{an+i}\frac{\partial}{\partial x_{an+i}}\qquad\rightarrow\qquad(6)$$

And the dot indicates the derivative with respect to time t. The vector field defined by

$$V_{J_6}=J_6(\xi)=X^i\frac{\partial}{\partial x_{6n+i}}-X^{n+i}\frac{\partial}{\partial x_{7n+i}}-X^{2n+i}\frac{\partial}{\partial x_{3n+i}}+X^{3n+i}\frac{\partial}{\partial x_{2n+i}}+$$
$$X^{4n+i}\frac{\partial}{\partial x_{5n+i}}-X^{5n+i}\frac{\partial}{\partial x_{4n+i}}-X^{6n+i}\frac{\partial}{\partial x_i}+X^{7n+i}\frac{\partial}{\partial x_{n+i}}\ \rightarrow\qquad(7)$$

Is named Liouvile vector field on Clifford $K\ddot{a}hler$ manifold (M,V).

The maps explained by $T\,,P\!:M\rightarrow R$ such that:

$$T=\frac{1}{2}m_i(\dot{x}_i^2+\dot{x}_{n+i}^2+\dot{x}_{2n+i}^2+\dot{x}_{3n+i}^2+\dot{x}_{4n+i}^2+\dot{x}_{5n+i}^2+\dot{x}_{6n+i}^2+\dot{x}_{7n+i}^2)$$
$$\therefore\ T=\frac{1}{2}m_i\sum_{a=0}^{7}\dot{x}_{an+i}^2\qquad,\quad P=m_igh$$

Are said to be the kinetic energy and the potential energy of the system, respectively. Here $m_i\,,g\ and\ h$ stand for mass of a mechanical system having m particles, the gravity acceleration and distance to the origin of a mechanical system on Clifford $K\ddot{a}hler$ manifold (M,V), respectively.

Then $L\!:M\rightarrow R$ is a map that satisfies the conditions:

i) $L=T-P$ is a Lagrangian function.
ii) the function given by $E_L^{J_6}=V_{J_6}(L)-L$, is energy function.

The operator i_{J_6} induced by J_6 and defined by:

$$i_{J_6}\omega(X_1,X_2,\ldots,X_r)=\sum_{i=1}^{r}\omega\,(X_1,\ldots,J_6X_i,\ldots,X_r)\qquad\rightarrow\qquad(8)$$

Is called vertical derivation, where $\omega\in\Lambda^r M$, $X_i\in\mathcal{X}(M)$. The vertical differentiation d_{J_6} is determined by:

$$d_{J_6}=\left[i_{J_6}\,,d\right]=i_{J_6}d-di_{J_6}\qquad\rightarrow\qquad(9)$$

Where d is the usual exterior derivation. We saw that the closed Clifford $Kähler$ form is the closed 2-form given by $\Phi_L^{J_6}=-dd_{J_6}L$ such that

$$d_{J_6}=\frac{\partial}{\partial x_{6n+i}}dx_i-\frac{\partial}{\partial x_{7n+i}}dx_{n+i}-\frac{\partial}{\partial x_{3n+i}}dx_{2n+i}+\frac{\partial}{\partial x_{2n+i}}dx_{3n+i}+\frac{\partial}{\partial x_{5n+i}}dx_{4n+i}-\frac{\partial}{\partial x_{4n+i}}dx_{5n+i}-\frac{\partial}{\partial x_i}dx_{6n+i}+\frac{\partial}{\partial x_{n+i}}dx_{7n+i}$$

Determined by operator:

$$d_{J_6}:\mathcal{F}(M)\rightarrow\Lambda^1M\qquad\rightarrow\qquad(10)$$

Then

$$\begin{aligned}
\Phi_L^{J_6}=&-\frac{\partial^2L}{\partial x_j\partial x_{6n+i}}dx_j\wedge dx_i+\frac{\partial^2L}{\partial x_j\partial x_{7n+i}}dx_j\wedge dx_{n+i}+\frac{\partial^2L}{\partial x_j\partial x_{3n+i}}dx_j\wedge dx_{2n+i}-\\
&\frac{\partial^2L}{\partial x_j\partial x_{2n+i}}dx_j\wedge dx_{3n+i}-\frac{\partial^2L}{\partial x_j\partial x_{5n+i}}dx_j\wedge dx_{4n+i}+\frac{\partial^2L}{\partial x_j\partial x_{4n+i}}dx_j\wedge dx_{5n+i}+\frac{\partial^2L}{\partial x_j\partial x_i}dx_j\wedge dx_{6n+i}\\
&-\frac{\partial^2L}{\partial x_j\partial x_{n+i}}dx_j\wedge dx_{7n+i}-\frac{\partial^2L}{\partial x_{n+j}\partial x_{6n+i}}dx_{n+j}\wedge dx_i+\frac{\partial^2L}{\partial x_{n+j}\partial x_{7n+i}}dx_{n+j}\wedge dx_{n+i}+\\
&+\frac{\partial^2L}{\partial x_{n+j}\partial x_{3n+i}}dx_{n+j}\wedge dx_{2n+i}-\frac{\partial^2L}{\partial x_{n+j}\partial x_{2n+i}}dx_{n+j}\wedge dx_{3n+i}-\frac{\partial^2L}{\partial x_{n+j}\partial x_{5n+i}}dx_{n+j}\wedge dx_{4n+i}\\
&+\frac{\partial^2L}{\partial x_{n+j}\partial x_{4n+i}}dx_{n+j}\wedge dx_{5n+i}+\frac{\partial^2L}{\partial x_{n+j}\partial x_i}dx_{n+j}\wedge dx_{6n+i}-\frac{\partial^2L}{\partial x_{n+j}\partial x_{n+i}}dx_{n+j}\wedge dx_{7n+i}\\
&-\frac{\partial^2L}{\partial x_{2n+j}\partial x_{6n+i}}dx_{2n+j}\wedge dx_i+\frac{\partial^2L}{\partial x_{2n+j}\partial x_{7n+i}}dx_{2n+j}\wedge dx_{n+i}+\frac{\partial^2L}{\partial x_{2n+j}\partial x_{3n+i}}dx_{2n+j}\wedge dx_{2n+i}\\
&-\frac{\partial^2L}{\partial x_{2n+j}\partial x_{2n+i}}dx_{2n+j}\wedge dx_{3n+i}-\frac{\partial^2L}{\partial x_{2n+j}\partial x_{5n+i}}dx_{2n+j}\wedge dx_{4n+i}+\frac{\partial^2L}{\partial x_{2n+j}\partial x_{4n+i}}dx_{2n+j}\wedge dx_{5n+i}+\\
&\frac{\partial^2L}{\partial x_{2n+j}\partial x_i}dx_{2n+j}\wedge dx_{6n+i}-\frac{\partial^2L}{\partial x_{2n+j}\partial x_{n+i}}dx_{2n+j}\wedge dx_{7n+i}-\frac{\partial^2L}{\partial x_{3n+j}\partial x_{6n+i}}dx_{3n+j}\wedge dx_i+\\
&\frac{\partial^2L}{\partial x_{3n+j}\partial x_{7n+i}}dx_{3n+j}\wedge dx_{n+i}+\frac{\partial^2L}{\partial x_{3n+j}\partial x_{3n+i}}dx_{3n+j}\wedge dx_{2n+i}-\frac{\partial^2L}{\partial x_{3n+j}\partial x_{2n+i}}dx_{3n+j}\wedge dx_{3n+i}-\\
&\frac{\partial^2L}{\partial x_{3n+j}\partial x_{5n+i}}dx_{3n+j}\wedge dx_{4n+i}+\frac{\partial^2L}{\partial x_{3n+j}\partial x_{4n+i}}dx_{3n+j}\wedge dx_{5n+i}+\frac{\partial^2L}{\partial x_{3n+j}\partial x_i}dx_{3n+j}\wedge dx_{6n+i}-\\
&\frac{\partial^2L}{\partial x_{3n+j}\partial x_{n+i}}dx_{3n+j}\wedge dx_{7n+i}-\frac{\partial^2L}{\partial x_{4n+j}\partial x_{6n+i}}dx_{4n+j}\wedge dx_i+\frac{\partial^2L}{\partial x_{4n+j}\partial x_{7n+i}}dx_{4n+j}\wedge dx_{n+i}+\\
&\frac{\partial^2L}{\partial x_{4n+j}\partial x_{3n+i}}dx_{4n+j}\wedge dx_{2n+i}-\frac{\partial^2L}{\partial x_{4n+j}\partial x_{2n+i}}dx_{4n+j}\wedge dx_{3n+i}-\frac{\partial^2L}{\partial x_{4n+j}\partial x_{5n+i}}dx_{4n+j}\wedge dx_{4n+i}+\\
&\frac{\partial^2L}{\partial x_{4n+j}\partial x_{4n+i}}dx_{4n+j}\wedge dx_{5n+i}+\frac{\partial^2L}{\partial x_{4n+j}\partial x_i}dx_{4n+j}\wedge dx_{6n+i}-\frac{\partial^2L}{\partial x_{4n+j}\partial x_{n+i}}dx_{4n+j}\wedge dx_{7n+i}-\\
&\frac{\partial^2L}{\partial x_{5n+j}\partial x_{6n+i}}dx_{5n+j}\wedge dx_i+\frac{\partial^2L}{\partial x_{5n+j}\partial x_{7n+i}}dx_{5n+j}\wedge dx_{n+i}+\frac{\partial^2L}{\partial x_{5n+j}\partial x_{3n+i}}dx_{5n+j}\wedge dx_{2n+i}-\\
&\frac{\partial^2L}{\partial x_{5n+j}\partial x_{2n+i}}dx_{5n+j}\wedge dx_{3n+i}-\frac{\partial^2L}{\partial x_{5n+j}\partial x_{5n+i}}dx_{5n+j}\wedge dx_{4n+i}+\frac{\partial^2L}{\partial x_{5n+j}\partial x_{4n+i}}dx_{5n+j}\wedge dx_{5n+i}+\\
&\frac{\partial^2L}{\partial x_{5n+j}\partial x_i}dx_{5n+j}\wedge dx_{6n+i}-\frac{\partial^2L}{\partial x_{5n+j}\partial x_{n+i}}dx_{5n+j}\wedge dx_{7n+i}-\frac{\partial^2L}{\partial x_{6n+j}\partial x_{6n+i}}dx_{6n+j}\wedge dx_i+\\
&\frac{\partial^2L}{\partial x_{6n+j}\partial x_{7n+i}}dx_{6n+j}\wedge dx_{n+i}+\frac{\partial^2L}{\partial x_{6n+j}\partial x_{3n+i}}dx_{6n+j}\wedge dx_{2n+i}-\frac{\partial^2L}{\partial x_{6n+j}\partial x_{2n+i}}dx_{6n+j}\wedge dx_{3n+i}-\\
&\frac{\partial^2L}{\partial x_{6n+j}\partial x_{5n+i}}dx_{6n+j}\wedge dx_{4n+i}+\frac{\partial^2L}{\partial x_{6n+j}\partial x_{4n+i}}dx_{6n+j}\wedge dx_{5n+i}+\frac{\partial^2L}{\partial x_{6n+j}\partial x_i}dx_{6n+j}\wedge dx_{6n+i}-\\
&\frac{\partial^2L}{\partial x_{6n+j}\partial x_{n+i}}dx_{6n+j}\wedge dx_{7n+i}-\frac{\partial^2L}{\partial x_{7n+j}\partial x_{6n+i}}dx_{7n+j}\wedge dx_i+\frac{\partial^2L}{\partial x_{7n+j}\partial x_{7n+i}}dx_{7n+j}\wedge dx_{n+i}+\\
&\frac{\partial^2L}{\partial x_{7n+j}\partial x_{3n+i}}dx_{7n+j}\wedge dx_{2n+i}-\frac{\partial^2L}{\partial x_{7n+j}\partial x_{2n+i}}dx_{7n+j}\wedge dx_{3n+i}-\frac{\partial^2L}{\partial x_{7n+j}\partial x_{5n+i}}dx_{7n+j}\wedge dx_{4n+i}+\\
&\frac{\partial^2L}{\partial x_{7n+j}\partial x_{4n+i}}dx_{7n+j}\wedge dx_{5n+i}+\frac{\partial^2L}{\partial x_{7n+j}\partial x_i}dx_{7n+j}\wedge dx_{6n+i}-\frac{\partial^2L}{\partial x_{7n+j}\partial x_{n+i}}dx_{7n+j}\wedge dx_{7n+i}
\end{aligned}$$

Let ξ be the second order differential equation by determined Eq(1) and given by Eq(5) and

$$i_\xi \Phi_L^{J_6} = -X^i \frac{\partial^2 L}{\partial x_j \partial x_{6n+i}} \delta_i^j dx_i + X^i \frac{\partial^2 L}{\partial x_j \partial x_{6n+i}} dx_j + X^i \frac{\partial^2 L}{\partial x_j \partial x_{7n+i}} \delta_i^j dx_{n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_j \partial x_{7n+i}} dx_j +$$

$$X^i \frac{\partial^2 L}{\partial x_j \partial x_{3n+i}} \delta_i^j dx_{2n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_j \partial x_{3n+i}} dx_j - X^i \frac{\partial^2 L}{\partial x_j \partial x_{2n+i}} \delta_i^j dx_{3n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_j \partial x_{2n+i}} dx_j -$$

$$X^i \frac{\partial^2 L}{\partial x_j \partial x_{5n+i}} \delta_i^j dx_{4n+i} + X^{4n+i} \frac{\partial^2 L}{\partial x_j \partial x_{5n+i}} dx_j + X^i \frac{\partial^2 L}{\partial x_j \partial x_{4n+i}} \delta_i^j dx_{5n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_j \partial x_{4n+i}} dx_j +$$

$$X^i \frac{\partial^2 L}{\partial x_j \partial x_i} \delta_i^j dx_{6n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_j \partial x_i} dx_j - X^i \frac{\partial^2 L}{\partial x_j \partial x_{n+i}} \delta_i^j dx_{7n+i} + X^{7n+i} \frac{\partial^2 L}{\partial x_j \partial x_{n+i}} dx_j -$$

$$X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{6n+i}} \delta_{n+i}^{n+j} dx_i + X^i \frac{\partial^2 L}{\partial x_{n+j} \partial x_{6n+i}} dx_{n+j} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{7n+i}} \delta_{n+i}^{n+j} dx_{n+i} -$$

$$X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{7n+i}} dx_{n+j} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{3n+i}} \delta_{n+i}^{n+j} dx_{2n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{3n+i}} dx_{n+j} -$$

$$X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}} \delta_{n+i}^{n+j} dx_{3n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}} dx_{n+j} - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{5n+i}} \delta_{n+i}^{n+j} dx_{4n+i} +$$

$$X^{4n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{5n+i}} dx_{n+j} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{4n+i}} \delta_{n+i}^{n+j} dx_{5n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{4n+i}} dx_{n+j} +$$

$$X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_i} \delta_{n+i}^{n+j} dx_{6n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_i} dx_{n+j} - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{n+i}} \delta_{n+i}^{n+j} dx_{7n+i} +$$

$$X^{7n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{n+i}} dx_{n+j} - X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{6n+i}} \delta_{2n+i}^{2n+j} dx_i + X^i \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{6n+i}} dx_{2n+j} +$$

$$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{7n+i}} \delta_{2n+i}^{2n+j} dx_{n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{7n+i}}$$

$$dx_{2n+j} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{3n+i}} \delta_{2n+i}^{2n+j} dx_{2n+i} -$$

$$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{3n+i}} dx_{2n+j} - X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{2n+i}} \delta_{2n+i}^{2n+j} dx_{3n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{2n+i}} dx_{2n+j} -$$

$$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{5n+i}} \delta_{2n+i}^{2n+j} dx_{4n+i} + X^{4n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{5n+i}} dx_{2n+j} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{4n+i}} \delta_{2n+i}^{2n+j} dx_{5n+i}$$

$$- X^{5n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{4n+i}} dx_{2n+j} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_i} \delta_{2n+i}^{2n+j} dx_{6n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_i} dx_{2n+j} -$$

$$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{n+i}} \delta_{2n+i}^{2n+j} dx_{7n+i} + X^{7n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{n+i}} dx_{2n+j} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{6n+i}} \delta_{3n+i}^{3n+j} dx_i +$$

$$X^i \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{6n+i}} dx_{3n+j} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{7n+i}} \delta_{3n+i}^{3n+j} dx_{n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{7n+i}} dx_{3n+j} +$$

$$X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{3n+i}} \delta_{3n+i}^{3n+j} dx_{2n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{3n+i}} dx_{3n+j} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{2n+i}} \delta_{3n+i}^{3n+j} dx_{3n+i}$$

$$+X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{2n+i}} dx_{3n+j} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{5n+i}} \delta_{3n+i}^{3n+j} dx_{4n+i} + X^{4n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{5n+i}} dx_{3n+j} +$$

$$X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{4n+i}} \delta_{3n+i}^{3n+j} dx_{5n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{4n+i}} dx_{3n+j} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_i} \delta_{3n+i}^{3n+j} dx_{6n+i} -$$

$$X^{6n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_i} dx_{3n+j} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{n+i}} \delta_{3n+i}^{3n+j} dx_{7n+i} + X^{7n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{n+i}} dx_{3n+j} -$$

$$X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{6n+i}} \delta_{4n+i}^{4n+j} dx_i + X^i \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{6n+i}} dx_{4n+j} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{7n+i}} \delta_{4n+i}^{4n+j} dx_{n+i} -$$

$$X^{n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{7n+i}} dx_{4n+j} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{3n+i}} \delta_{4n+i}^{4n+j} dx_{2n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{3n+i}} dx_{4n+j} -$$

$$X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{2n+i}} \delta_{4n+i}^{4n+j} dx_{3n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{2n+i}} dx_{4n+j} - X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{5n+i}} \delta_{4n+i}^{4n+j} dx_{4n+i}$$

$$+X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{5n+i}} dx_{4n+j} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{4n+i}} \delta_{4n+i}^{4n+j} dx_{5n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{4n+i}} dx_{4n+j} +$$

$$X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_i} \delta_{4n+i}^{4n+j} dx_{6n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_i} dx_{4n+j} - X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{n+i}} \delta_{4n+i}^{4n+j} dx_{7n+i} +$$

$$X^{7n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{n+i}} dx_{4n+j} - X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{6n+i}} \delta_{5n+i}^{5n+j} dx_i + X^i \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{6n+i}} dx_{5n+j} +$$

$$X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{7n+i}} \delta_{5n+i}^{5n+j} dx_{n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{7n+i}} dx_{5n+j} + X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{3n+i}} \delta_{5n+i}^{5n+j} dx_{2n+i} -$$

$$X^{2n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{3n+i}} dx_{5n+j} - X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{2n+i}} \delta_{5n+i}^{5n+j} dx_{3n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{2n+i}} dx_{5n+j} -$$

$$X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{5n+i}} \delta_{5n+i}^{5n+j} dx_{4n+i} + X^{4n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{5n+i}} dx_{5n+j} + X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{4n+i}} \delta_{5n+i}^{5n+j} dx_{5n+i}$$

$$-X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{4n+i}} dx_{5n+j} + X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_i} \delta_{5n+i}^{5n+j} dx_{6n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_i} dx_{5n+j} -$$

$$X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{n+i}} \delta_{5n+i}^{5n+j} dx_{7n+i} + X^{7n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{n+i}} dx_{5n+j} - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{6n+i}} \delta_{6n+i}^{6n+j} dx_i$$

$$+X^i \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{6n+i}} dx_{6n+j} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{7n+i}} \delta_{6n+i}^{6n+j} dx_{n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{7n+i}} dx_{6n+j} +$$

$$X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{3n+i}} \delta_{6n+i}^{6n+j} dx_{2n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{3n+i}} dx_{6n+j} - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{2n+i}} \delta_{6n+i}^{6n+j} dx_{3n+i}$$

$$
\begin{aligned}
&+X^{3n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{2n+i}}dx_{6n+j}-X^{6n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{5n+i}}\delta_{6n+i}^{6n+j}dx_{4n+i}+X^{4n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{5n+i}}dx_{6n+j}+\\
&X^{6n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{4n+i}}\delta_{6n+i}^{6n+j}dx_{5n+i}-X^{5n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{4n+i}}dx_{6n+j}+X^{6n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{i}}\delta_{6n+i}^{6n+j}dx_{6n+i}\\
&-X^{6n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{i}}dx_{6n+j}-X^{6n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{n+i}}\delta_{6n+i}^{6n+j}dx_{7n+i}+X^{7n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{n+i}}dx_{6n+j}-\\
&X^{7n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{6n+i}}\delta_{7n+i}^{7n+j}dx_{i}+X^{i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{6n+i}}dx_{7n+j}+X^{7n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{7n+i}}\delta_{7n+i}^{7n+j}dx_{n+i}-\\
&X^{n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{7n+i}}dx_{7n+j}+X^{7n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{3n+i}}\delta_{7n+i}^{7n+j}dx_{2n+i}-X^{2n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{3n+i}}dx_{7n+j}-\\
&X^{7n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{2n+i}}\delta_{7n+i}^{7n+j}dx_{3n+i}+X^{3n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{2n+i}}dx_{7n+j}-X^{7n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{5n+i}}\delta_{7n+i}^{7n+j}dx_{4n+i}\\
&+X^{4n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{5n+i}}dx_{7n+j}+X^{7n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{4n+i}}\delta_{7n+i}^{7n+j}dx_{5n+i}-X^{5n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{4n+i}}dx_{7n+j}+\\
&X^{7n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{i}}\delta_{7n+i}^{7n+j}dx_{6n+i}-X^{6n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{i}}dx_{7n+j}-X^{7n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{n+i}}\delta_{7n+i}^{7n+j}dx_{7n+i}\\
&+X^{7n+i}\frac{\partial^{2}L}{\partial x_{7n+j}\partial x_{n+i}}dx_{7n+j}
\end{aligned}
$$

Since the closed Clifford $K\ddot{a}hler$ form $\Phi_{L}^{J_6}$ $on\ M$ is the symplectic structure

$$
\begin{aligned}
E_{L}^{J_6}=V_{J_6}(L)-L=X^{i}\frac{\partial L}{\partial x_{6n+i}}-X^{n+i}\frac{\partial L}{\partial x_{7n+i}}-X^{2n+i}\frac{\partial L}{\partial x_{3n+i}}+X^{3n+i}\frac{\partial L}{\partial x_{2n+i}}\\
+X^{4n+i}\frac{\partial L}{\partial x_{5n+i}}-X^{5n+i}\frac{\partial L}{\partial x_{4n+i}}-X^{6n+i}\frac{\partial L}{\partial x_{i}}+X^{7n+i}\frac{\partial L}{\partial x_{n+i}}-L\quad\rightarrow
\end{aligned}
\tag{11}
$$

And thus

$$
\begin{aligned}
dE_{L}^{J_6}=&X^{i}\frac{\partial^{2}L}{\partial x_{j}\partial x_{6n+i}}dx_{j}-X^{n+i}\frac{\partial^{2}L}{\partial x_{j}\partial x_{7n+i}}dx_{j}-X^{2n+i}\frac{\partial^{2}L}{\partial x_{j}\partial x_{3n+i}}dx_{j}+X^{3n+i}\frac{\partial^{2}L}{\partial x_{j}\partial x_{2n+i}}dx_{j}\\
&+X^{4n+i}\frac{\partial^{2}L}{\partial x_{j}\partial x_{5n+i}}dx_{j}-X^{5n+i}\frac{\partial^{2}L}{\partial x_{4n+j}\partial x_{6n+i}}dx_{j}-X^{6n+i}\frac{\partial^{2}L}{\partial x_{j}\partial x_{i}}dx_{j}+X^{7n+i}\frac{\partial^{2}L}{\partial x_{j}\partial x_{n+i}}dx_{j}\\
&+X^{i}\frac{\partial^{2}L}{\partial x_{n+j}\partial x_{6n+i}}dx_{n+j}-X^{n+i}\frac{\partial^{2}L}{\partial x_{n+j}\partial x_{7n+i}}dx_{n+j}-X^{2n+i}\frac{\partial^{2}L}{\partial x_{n+j}\partial x_{3n+i}}dx_{n+j}+\\
&+X^{3n+i}\frac{\partial^{2}L}{\partial x_{n+j}\partial x_{2n+i}}dx_{n+j}+X^{4n+i}\frac{\partial^{2}L}{\partial x_{n+j}\partial x_{5n+i}}dx_{n+j}-X^{5n+i}\frac{\partial^{2}L}{\partial x_{n+j}\partial x_{4n+i}}dx_{n+j}-\\
&X^{6n+i}\frac{\partial^{2}L}{\partial x_{n+j}\partial x_{i}}dx_{n+j}+X^{7n+i}\frac{\partial^{2}L}{\partial x_{n+j}\partial x_{n+i}}dx_{n+j}+X^{i}\frac{\partial^{2}L}{\partial x_{2n+j}\partial x_{6n+i}}dx_{2n+j}-\\
&X^{n+i}\frac{\partial^{2}L}{\partial x_{2n+j}\partial x_{7n+i}}dx_{2n+j}-X^{2n+i}\frac{\partial^{2}L}{\partial x_{2n+j}\partial x_{3n+i}}dx_{2n+j}+X^{3n+i}\frac{\partial^{2}L}{\partial x_{2n+j}\partial x_{2n+i}}dx_{2n+j}+\\
&X^{4n+i}\frac{\partial^{2}L}{\partial x_{2n+j}\partial x_{5n+i}}dx_{2n+j}-X^{5n+i}\frac{\partial^{2}L}{\partial x_{2n+j}\partial x_{4n+i}}dx_{2n+j}-X^{6n+i}\frac{\partial^{2}L}{\partial x_{2n+j}\partial x_{i}}dx_{2n+j}+\\
&X^{7n+i}\frac{\partial^{2}L}{\partial x_{2n+j}\partial x_{n+i}}dx_{2n+j}+X^{i}\frac{\partial^{2}L}{\partial x_{3n+j}\partial x_{6n+i}}dx_{3n+j}-X^{n+i}\frac{\partial^{2}L}{\partial x_{3n+j}\partial x_{7n+i}}dx_{3n+j}-\\
&X^{2n+i}\frac{\partial^{2}L}{\partial x_{3n+j}\partial x_{3n+i}}dx_{3n+j}+X^{3n+i}\frac{\partial^{2}L}{\partial x_{3n+j}\partial x_{2n+i}}dx_{3n+j}+X^{4n+i}\frac{\partial^{2}L}{\partial x_{3n+j}\partial x_{5n+i}}dx_{3n+j}-\\
&X^{5n+i}\frac{\partial^{2}L}{\partial x_{3n+j}\partial x_{4n+i}}dx_{3n+j}-X^{6n+i}\frac{\partial^{2}L}{\partial x_{3n+j}\partial x_{i}}dx_{3n+j}+X^{7n+i}\frac{\partial^{2}L}{\partial x_{3n+j}\partial x_{n+i}}dx_{3n+j}+\\
&X^{i}\frac{\partial^{2}L}{\partial x_{4n+j}\partial x_{6n+i}}dx_{4n+j}-X^{n+i}\frac{\partial^{2}L}{\partial x_{4n+j}\partial x_{7n+i}}dx_{4n+j}-X^{2n+i}\frac{\partial^{2}L}{\partial x_{4n+j}\partial x_{3n+i}}dx_{4n+j}+\\
&X^{3n+i}\frac{\partial^{2}L}{\partial x_{4n+j}\partial x_{2n+i}}dx_{4n+j}+X^{4n+i}\frac{\partial^{2}L}{\partial x_{4n+j}\partial x_{5n+i}}dx_{4n+j}-X^{5n+i}\frac{\partial^{2}L}{\partial x_{4n+j}\partial x_{4n+i}}dx_{4n+j}-\\
&X^{6n+i}\frac{\partial^{2}L}{\partial x_{4n+j}\partial x_{i}}dx_{4n+j}+X^{7n+i}\frac{\partial^{2}L}{\partial x_{4n+j}\partial x_{n+i}}dx_{4n+j}+X^{i}\frac{\partial^{2}L}{\partial x_{5n+j}\partial x_{6n+i}}dx_{5n+j}-\\
&X^{n+i}\frac{\partial^{2}L}{\partial x_{5n+j}\partial x_{7n+i}}dx_{5n+j}-X^{2n+i}\frac{\partial^{2}L}{\partial x_{5n+j}\partial x_{3n+i}}dx_{5n+j}+X^{3n+i}\frac{\partial^{2}L}{\partial x_{5n+j}\partial x_{2n+i}}dx_{5n+j}+\\
&X^{4n+i}\frac{\partial^{2}L}{\partial x_{5n+j}\partial x_{5n+i}}dx_{5n+j}-X^{5n+i}\frac{\partial^{2}L}{\partial x_{5n+j}\partial x_{4n+i}}dx_{5n+j}-X^{6n+i}\frac{\partial^{2}L}{\partial x_{5n+j}\partial x_{i}}dx_{5n+j}+\\
&X^{7n+i}\frac{\partial^{2}L}{\partial x_{5n+j}\partial x_{n+i}}dx_{5n+j}+X^{i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{6n+i}}dx_{6n+j}-X^{n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{7n+i}}dx_{6n+j}-\\
&X^{2n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{3n+i}}dx_{6n+j}+X^{3n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{2n+i}}dx_{6n+j}+X^{4n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{5n+i}}dx_{6n+j}-\\
&X^{5n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{4n+i}}dx_{6n+j}-X^{6n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{i}}dx_{6n+j}+X^{7n+i}\frac{\partial^{2}L}{\partial x_{6n+j}\partial x_{n+i}}dx_{6n+j}+
\end{aligned}
$$

$X^i \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}} dx_{7n+j} - X^{n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}} dx_{7n+j} - X^{2n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}} dx_{7n+j} + X^{3n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}} dx_{7n+j} + X^{4n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}} dx_{7n+j} - X^{5n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}} dx_{7n+j}$
$-X^{6n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_i} dx_{7n+j} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}} dx_{7n+j} - \frac{\partial L}{\partial x_j} dx_j - \frac{\partial L}{\partial x_{n+j}} dx_{n+j} - \frac{\partial L}{\partial x_{2n+j}} dx_{2n+j}$
$- \frac{\partial L}{\partial x_{3n+j}} dx_{3n+j} - \frac{\partial L}{\partial x_{4n+j}} dx_{4n+j} - \frac{\partial L}{\partial x_{5n+j}} dx_{5n+j} - \frac{\partial L}{\partial x_{6n+j}} dx_{6n+j} - \frac{\partial L}{\partial x_{7n+j}} dx_{7n+j}.$

By means of Eq(1), we calculate the following expressions.

$-X^i \frac{\partial^2 L}{\partial x_j\partial x_{6n+i}} \delta_i^j dx_i + X^i \frac{\partial^2 L}{\partial x_j\partial x_{7n+i}} \delta_i^j dx_{n+i} + X^i \frac{\partial^2 L}{\partial x_j\partial x_{3n+i}} \delta_i^j dx_{2n+i} - X^i \frac{\partial^2 L}{\partial x_j\partial x_{2n+i}} \delta_i^j dx_{3n+i} -$

$-X^i \frac{\partial^2 L}{\partial x_j\partial x_{5n+i}} \delta_i^j dx_{4n+i} + X^i \frac{\partial^2 L}{\partial x_j\partial x_{4n+i}} \delta_i^j dx_{5n+i} + X^i \frac{\partial^2 L}{\partial x_j\partial x_i} \delta_i^j dx_{6n+i} - X^i \frac{\partial^2 L}{\partial x_j\partial x_{n+i}} \delta_i^j dx_{7n+i}$
$-X^{n+i} \frac{\partial^2 L}{\partial x_{n+j}\partial x_{6n+i}} \delta_{n+i}^{n+j} dx_i + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j}\partial x_{7n+i}} \delta_{n+i}^{n+j} dx_{n+i} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j}\partial x_{3n+i}} \delta_{n+i}^{n+j} dx_{2n+i} -$
$X^{n+i} \frac{\partial^2 L}{\partial x_{n+j}\partial x_{2n+i}} \delta_{n+i}^{n+j} dx_{3n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j}\partial x_{5n+i}} \delta_{n+i}^{n+j} dx_{4n+i} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j}\partial x_{4n+i}} \delta_{n+i}^{n+j} dx_{5n+i} +$
$X^{n+i} \frac{\partial^2 L}{\partial x_{n+j}\partial x_i} \delta_{n+i}^{n+j} dx_{6n+i} - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j}\partial x_{n+i}} \delta_{n+i}^{n+j} dx_{7n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j}\partial x_{6n+i}} \delta_{2n+i}^{2n+j} dx_i +$
$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j}\partial x_{7n+i}} \delta_{2n+i}^{2n+j} dx_{n+i} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j}\partial x_{3n+i}} \delta_{2n+i}^{2n+j} dx_{2n+i} -$
$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j}\partial x_{2n+i}} \delta_{2n+i}^{2n+j} dx_{3n+i} - X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j}\partial x_{5n+i}} \delta_{2n+i}^{2n+j} dx_{4n+i} +$
$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j}\partial x_{4n+i}} \delta_{2n+i}^{2n+j} dx_{5n+i} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j}\partial x_i} \delta_{2n+i}^{2n+j} dx_{6n+i} -$
$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j}\partial x_{n+i}} \delta_{2n+i}^{2n+j} dx_{7n+i} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j}\partial x_{6n+i}} \delta_{3n+i}^{3n+j} dx_i +$
$X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j}\partial x_{7n+i}} \delta_{3n+i}^{3n+j} dx_{n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j}\partial x_{3n+i}} \delta_{3n+i}^{3n+j} dx_{2n+i} -$
$X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j}\partial x_{2n+i}} \delta_{3n+i}^{3n+j} dx_{3n+i} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j}\partial x_{5n+i}} \delta_{3n+i}^{3n+j} dx_{4n+i} +$
$X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j}\partial x_{4n+i}} \delta_{3n+i}^{3n+j} dx_{5n+i} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j}\partial x_i} \delta_{3n+i}^{3n+j} dx_{6n+i} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j}\partial x_{n+i}} \delta_{3n+i}^{3n+j} dx_{7n+i}$
$- X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j}\partial x_{6n+i}} \delta_{4n+i}^{4n+j} dx_i + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j}\partial x_{7n+i}} \delta_{4n+i}^{4n+j} dx_{n+i} +$
$X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j}\partial x_{3n+i}} \delta_{4n+i}^{4n+j} dx_{2n+i} - X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j}\partial x_{2n+i}} \delta_{4n+i}^{4n+j} dx_{3n+i} -$
$X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j}\partial x_{5n+i}} \delta_{4n+i}^{4n+j} dx_{4n+i} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j}\partial x_{4n+i}} \delta_{4n+i}^{4n+j} dx_{5n+i} +$
$X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j}\partial x_i} \delta_{4n+i}^{4n+j} dx_{6n+i} - X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j}\partial x_{n+i}} \delta_{4n+i}^{4n+j} dx_{7n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j}\partial x_{6n+i}} \delta_{5n+i}^{5n+j} dx_i$
$+ X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j}\partial x_{7n+i}} \delta_{5n+i}^{5n+j} dx_{n+i} + X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j}\partial x_{3n+i}} \delta_{5n+i}^{5n+j} dx_{2n+i}$
$- X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j}\partial x_{2n+i}} \delta_{5n+i}^{5n+j} dx_{3n+i} - X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j}\partial x_{5n+i}} \delta_{5n+i}^{5n+j} dx_{4n+i} +$
$X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j}\partial x_{4n+i}} \delta_{5n+i}^{5n+j} dx_{5n+i} + X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j}\partial x_i} \delta_{5n+i}^{5n+j} dx_{6n+i} -$
$X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j}\partial x_{n+i}} \delta_{5n+i}^{5n+j} dx_{7n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j}\partial x_{6n+i}} \delta_{6n+i}^{6n+j} dx_i + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j}\partial x_{7n+i}} \delta_{6n+i}^{6n+j} dx_{n+i}$
$+ X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j}\partial x_{3n+i}} \delta_{6n+i}^{6n+j} dx_{2n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j}\partial x_{2n+i}} \delta_{6n+i}^{6n+j} dx_{3n+i} -$
$X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j}\partial x_{5n+i}} \delta_{6n+i}^{6n+j} dx_{4n+i} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j}\partial x_{4n+i}} \delta_{6n+i}^{6n+j} dx_{5n+i} +$
$X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j}\partial x_i} \delta_{6n+i}^{6n+j} dx_{6n+i} - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j}\partial x_{n+i}} \delta_{6n+i}^{6n+j} dx_{7n+i} -$
$X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{6n+i}} \delta_{7n+i}^{7n+j} dx_i + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{7n+i}} \delta_{7n+i}^{7n+j} dx_{n+i} +$
$X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{3n+i}} \delta_{7n+i}^{7n+j} dx_{2n+i} - X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{2n+i}} \delta_{7n+i}^{7n+j} dx_{3n+i} -$
$X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{5n+i}} \delta_{7n+i}^{7n+j} dx_{4n+i} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{4n+i}} \delta_{7n+i}^{7n+j} dx_{5n+i} +$
$X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_i} \delta_{6n+i}^{6n+j} dx_{6n+i} - X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j}\partial x_{n+i}} \delta_{7n+i}^{7n+j} dx_{7n+i} + \frac{\partial L}{\partial x_j} dx_j + \frac{\partial L}{\partial x_{n+j}} dx_{n+j}$
$+ \frac{\partial L}{\partial x_{2n+j}} dx_{2n+j} + \frac{\partial L}{\partial x_{3n+j}} dx_{3n+j} + \frac{\partial L}{\partial x_{4n+j}} dx_{4n+j} + \frac{\partial L}{\partial x_{5n+j}} dx_{5n+j} + \frac{\partial L}{\partial x_{6n+j}} dx_{6n+j} +$

$\frac{\partial L}{\partial x_{7n+j}} dx_{7n+j} = 0$

If a curve determined by $\alpha: R \to M$ is taken to be an integral curve of ξ, then we found equation as follows:

$$-X^i \frac{\partial^2 L}{\partial x_j \partial x_{6n+i}} dx_j - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{6n+i}} dx_j - X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{6n+i}} dx_j - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{6n+i}} dx_j$$
$$-X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{6n+i}} dx_j - X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{6n+i}} dx_j - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{6n+i}} dx_j -$$
$$X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{6n+i}} dx_j + X^i \frac{\partial^2 L}{\partial x_j \partial x_{7n+i}} dx_{n+j} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{7n+i}} dx_{n+j} +$$
$$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{7n+i}} dx_{n+j} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{7n+i}} dx_{n+j} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{7n+i}} dx_{n+j} +$$
$$X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{7n+i}} dx_{n+j} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{7n+i}} dx_{n+j} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{7n+i}} dx_{n+j} +$$
$$X^i \frac{\partial^2 L}{\partial x_j \partial x_{3n+i}} dx_{2n+j} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{3n+i}} dx_{2n+j} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{3n+i}} dx_{2n+j} +$$
$$X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{3n+i}} dx_{2n+j} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{3n+i}} dx_{2n+j} + X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{3n+i}} dx_{2n+j} +$$
$$X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{3n+i}} dx_{2n+j} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{3n+i}} dx_{2n+j} - X^i \frac{\partial^2 L}{\partial x_j \partial x_{2n+i}} dx_{3n+j} -$$
$$X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}} dx_{3n+j} - X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{2n+i}} dx_{3n+j} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{2n+i}} dx_{3n+j} -$$
$$X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{2n+i}} dx_{3n+j} - X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{2n+i}} dx_{3n+j} - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{2n+i}} dx_{3n+j} -$$
$$X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{2n+i}} dx_{3n+j} - X^i \frac{\partial^2 L}{\partial x_j \partial x_{5n+i}} dx_{4n+j} - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{5n+i}} dx_{4n+j} -$$
$$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{5n+i}} dx_{4n+j} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{5n+i}} dx_{4n+j} - X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{5n+i}} dx_{4n+j} -$$
$$X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{5n+i}} dx_{4n+j} - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{5n+i}} dx_{4n+j} - X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{5n+i}} dx_{4n+j} +$$
$$X^i \frac{\partial^2 L}{\partial x_j \partial x_{4n+i}} dx_{5n+j} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{4n+i}} dx_{5n+j} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{4n+i}} dx_{5n+j} +$$
$$X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{4n+i}} dx_{5n+j} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{4n+i}} dx_{5n+j} + X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{4n+i}} dx_{5n+j} +$$
$$X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{4n+i}} dx_{5n+j} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{4n+i}} dx_{5n+j} + X^i \frac{\partial^2 L}{\partial x_j \partial x_i} dx_{6n+j} +$$
$$X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_i} dx_{6n+j} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_i} dx_{6n+j} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_i} dx_{6n+j} +$$
$$X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_i} dx_{6n+j} + X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_i} dx_{6n+j} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_i} dx_{6n+j} +$$
$$X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_i} dx_{6n+j} - X^i \frac{\partial^2 L}{\partial x_j \partial x_{n+i}} dx_{7n+j} - X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{n+i}} dx_{7n+j} -$$
$$X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{n+i}} dx_{7n+j} - X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{n+i}} dx_{7n+j} - X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{n+i}} dx_{7n+j} -$$
$$X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{n+i}} dx_{7n+j} - X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{n+i}} dx_{7n+j} - X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{n+i}} dx_{7n+j} + \frac{\partial L}{\partial x_j} dx_j +$$
$$\frac{\partial L}{\partial x_{n+j}} dx_{n+j} + \frac{\partial L}{\partial x_{2n+j}} dx_{2n+j} + \frac{\partial L}{\partial x_{3n+j}} dx_{3n+j} + \frac{\partial L}{\partial x_{4n+j}} dx_{4n+j} + \frac{\partial L}{\partial x_{5n+j}} dx_{5n+j} +$$
$$\frac{\partial L}{\partial x_{6n+j}} dx_{6n+j} + \frac{\partial L}{\partial x_{7n+j}} dx_{7n+j} = 0 \quad \rightarrow \quad (12)$$

Or

$$-[X^i \frac{\partial^2 L}{\partial x_j \partial x_{6n+i}} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{6n+i}} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{6n+i}} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{6n+i}} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{6n+i}}$$
$$+ X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{6n+i}} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{6n+i}} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{6n+i}}] dx_j + \frac{\partial L}{\partial x_j} dx_j +$$
$$[X^i \frac{\partial^2 L}{\partial x_j \partial x_{7n+i}} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{7n+i}} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{7n+i}} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{7n+i}} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{7n+i}}$$
$$+ X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{7n+i}} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{7n+i}} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{7n+i}}] dx_{n+j} + \frac{\partial L}{\partial x_{n+j}} dx_{n+j} +$$
$$[X^i \frac{\partial^2 L}{\partial x_j \partial x_{3n+i}} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{3n+i}} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{3n+i}} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{3n+i}} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{3n+i}}$$
$$+ X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{3n+i}} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{3n+i}} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{3n+i}}] dx_{2n+j} + \frac{\partial L}{\partial x_{2n+j}} dx_{2n+j} -$$

$$[X^i \frac{\partial^2 L}{\partial x_j \partial x_{2n+i}} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{2n+i}} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{2n+i}} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{2n+i}} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{2n+i}}$$
$$+ X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{2n+i}} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{2n+i}} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{2n+i}}]\, dx_{3n+j} + \frac{\partial L}{\partial x_{3n+j}} dx_{3n+j} -$$
$$[X^i \frac{\partial^2 L}{\partial x_j \partial x_{5n+i}} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{5n+i}} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{5n+i}} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{5n+i}} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{5n+i}}$$
$$+ X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{5n+i}} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{5n+i}} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{5n+i}}]\, dx_{4n+j} + \frac{\partial L}{\partial x_{4n+j}} dx_{4n+j} +$$
$$[X^i \frac{\partial^2 L}{\partial x_j \partial x_{4n+i}} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{4n+i}} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{4n+i}} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{4n+i}} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{4n+i}}$$
$$+ X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{4n+i}} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{4n+i}} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{4n+i}}]\, dx_{5n+j} + \frac{\partial L}{\partial x_{5n+j}} dx_{5n+j} +$$
$$[X^i \frac{\partial^2 L}{\partial x_j \partial x_i} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_i} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_i} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_i} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_i} +$$
$$X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_i} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_i} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_i}]\, dx_{6n+j} + \frac{\partial L}{\partial x_{6n+j}} dx_{6n+j} -$$
$$[X^i \frac{\partial^2 L}{\partial x_j \partial x_{n+i}} + X^{n+i} \frac{\partial^2 L}{\partial x_{n+j} \partial x_{n+i}} + X^{2n+i} \frac{\partial^2 L}{\partial x_{2n+j} \partial x_{n+i}} + X^{3n+i} \frac{\partial^2 L}{\partial x_{3n+j} \partial x_{n+i}} + X^{4n+i} \frac{\partial^2 L}{\partial x_{4n+j} \partial x_{n+i}}$$
$$+ X^{5n+i} \frac{\partial^2 L}{\partial x_{5n+j} \partial x_{n+i}} + X^{6n+i} \frac{\partial^2 L}{\partial x_{6n+j} \partial x_{n+i}} + X^{7n+i} \frac{\partial^2 L}{\partial x_{7n+j} \partial x_{n+i}}]\, dx_{7n+j} + \frac{\partial L}{\partial x_{7n+j}} dx_{7n+j} = 0$$

In this equation can be concise manner.

$$- \sum_{a=0}^{7} X^{an+i} \frac{\partial^2 L}{\partial x_{an+j} \partial x_{6n+i}} dx_j + \frac{\partial L}{\partial x_j} dx_j + \sum_{a=0}^{7} X^{an+i} \frac{\partial^2 L}{\partial x_{an+j} \partial x_{7n+i}} dx_{n+j} + \frac{\partial L}{\partial x_{n+j}} dx_{n+j} +$$
$$\sum_{a=0}^{7} X^{an+i} \frac{\partial^2 L}{\partial x_{an+j} \partial x_{3n+i}} dx_{2n+j} + \frac{\partial L}{\partial x_{2n+j}} dx_{2n+j} - \sum_{a=0}^{7} X^{an+i} \frac{\partial^2 L}{\partial x_{an+j} \partial x_{2n+i}} dx_{3n+j} +$$
$$\frac{\partial L}{\partial x_{3n+j}} dx_{3n+j} - \sum_{a=0}^{7} X^{an+i} \frac{\partial^2 L}{\partial x_{an+j} \partial x_{5n+i}} dx_{4n+j} + \frac{\partial L}{\partial x_{4n+j}} dx_{4n+j} +$$
$$\sum_{a=0}^{7} X^{an+i} \frac{\partial^2 L}{\partial x_{an+j} \partial x_{4n+i}} dx_{5n+j} + \frac{\partial L}{\partial x_{5n+j}} dx_{5n+j} + \sum_{a=0}^{7} X^{an+i} \frac{\partial^2 L}{\partial x_{an+j} \partial x_i} dx_{6n+j} + \frac{\partial L}{\partial x_{6n+j}} dx_{6n+j}$$
$$- \sum_{a=0}^{7} X^{an+i} \frac{\partial^2 L}{\partial x_{an+j} \partial x_{n+i}} dx_{7n+j} + \frac{\partial L}{\partial x_{7n+j}} dx_{7n+j} = 0 \quad \rightarrow \quad (13)$$

Then we find the equations

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_i}\right) + \frac{\partial L}{\partial x_{6n+i}} = 0, \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{n+i}}\right) - \frac{\partial L}{\partial x_{7n+i}} = 0, \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{2n+i}}\right) - \frac{\partial L}{\partial x_{3n+i}} = 0,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{3n+i}}\right) + \frac{\partial L}{\partial x_{2n+i}} = 0, \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{4n+i}}\right) + \frac{\partial L}{\partial x_{5n+i}} = 0, \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{5n+i}}\right) - \frac{\partial L}{\partial x_{4n+i}} = 0,$$
$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{6n+i}}\right) + \frac{\partial L}{\partial x_i} = 0, \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_{7n+i}}\right) + \frac{\partial L}{\partial x_{n+i}} = 0 \quad \rightarrow \quad (14)$$

Thus equations obtained in Eq(14) infer Euler-Lagrange equations structured by means of $\Phi_L^{J_6}$ on Clifford $K\ddot{a}hler$ manifold (M, V) and so, the triple $(M, \Phi_L^{J_6}, \xi)$ is called a mechanical system on Clifford $K\ddot{a}hler$ manifold (M, V).

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