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RESEARCH ARTICLE

DERIVING RECOGNIZABLE NUMBERS USING INVERSE Z -TRANSFORMS

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ABSTRACT

In this communication, we determine some recognizable numbers by applying the properties of inverse Z -Transforms.

INTRODUCTION

Z -Transform is a Transform for a sequence. The development of communication branch is based on discrete analysis. Z -Transform plays the same role in discrete analysis as in continuous systems. The Z -Transform technique used for time signals and systems. In this communication, we develop some familiar numbers by using the properties of inverse Z - transforms, the partial fraction method, the residues theorem and convolution theorem.

Notations

$J_n \,:\, \frac{1}{3}\left[2^n - (-1)^n\right] =$ Jacobsthal number of rank n

$K_n \,:\, 4^n + 2^{n+1} - 1 =$ Kynea number of rank n

$T_n \,: 3 \cdot 2^n - 1 =$ Thabit number of rank n

$Carl_n \,: 4^n - 2^{n+1} - 1 =$ Carlol number of rank n

$W_n \,: n\; 2^n - 1 =$ Woodall number of rank n

$C_n \,:\, n\; 2^n + 1 =$ Cullen number of rank n

Definition

If the function u_n is defined for discrete values $(n = 0\,, 1, 2, 3, ...)$ and $u_n = 0 \;\; for \;\; n < 0$, then its Z - Transforms is defined to be $Z(u_n) = U(z) = \sum u_n\, z^{-n}$. The inverse Z -Transform is written as $Z^{-1}[U(z)] = u_n$.

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Theorem: I

The inverse Z -Transform of $U(z)$ is given by the formula $u_n = \frac{1}{2\pi i}\int_c U(z)z^{n-1}dz =$ sum of residues of $U(z)z^{n-1}$ at the poles of $U(z)$ which are inside the contour C.

Theorem: II

If $Z^{-1}[U(z)] = u_n \ and \ Z^{-1}[V(z)] = v_n$, then $Z^{-1}[U(z)\cdot V(z)] = \sum u_m \ v_{n-m} = u_n * v_n$ where the symbol denotes the convolution operation.

Method of Analysis

The procedure for determining the sequence of some special numbers by applying inverse Z -Transform is explained detailly as below.

Theorem 1

$$Z^{-1}\left[\frac{z}{(z+1)(z-2)}\right] = J_n \text{ where } |z| < 3$$

Proof

Take $U(z) = \frac{z}{(z+1)(z-2)}$

The poles of $U(z)$ are $z = -1, z = 2$

Using $U(z)$ in the inversion integral, we have

$u_n = Z^{-1}[U(z)] =$ Sum of the residues of $U(z)\cdot z^{n-1}$ at the poles $z = -1, z = 2$.

Now,

$$\text{Res}\left[U(z)\cdot z^{n-1}\right]_{z=-1} = \underset{z\to -1}{Lt}\left[\frac{z^n}{(z-2)}\right] = \frac{(-1)^n}{-3} = \frac{-(-1)^n}{3}$$

$$\text{Res}\left[U(z)\cdot z^{n-1}\right]_{z=2} = \underset{z\to 2}{Lt}\left[\frac{z^n}{(z+1)}\right] = \frac{2^n}{3}$$

In view of theorem (I), we get

$$Z^{-1}\left[\frac{z}{(z+1)(z-2)}\right] = \frac{2^n}{3} - \frac{(-1)^n}{3} = \frac{1}{3}\left[2^n - (-1)^n\right] = J_n$$

Theorem: 2

$$Z^{-1}\left[\frac{2z^3 - 7z^2 + 2z}{(z-1)(z-2)(z-4)}\right] = K_n \text{ where } |z| < 5$$

Proof

Choose $U(z) = \left[\frac{2z^3 - 7z^2 + 2z}{(z-1)(z-2)(z-4)}\right]$

The poles of $U(z)$ are $z=1, z=2$ and $z=4$

By theorem I, we have

$Z^{-1}[U(z)]=$ Sum of the residues of $U(z)\cdot z^{n-1}$ at the poles are $z=1, z=2, z=4$.

Now,

Res $\left[U(z)\cdot z^{n-1}\right]_{z=1} = \underset{z\to 1}{Lt}\left[\frac{2z^{n+2}-7z^{n+1}+2z^{n}}{(z-2)(z-4)}\right]=-1$

Res $\left[U(z)\cdot z^{n-1}\right]_{z=2} = \underset{z\to 2}{Lt}\left[\frac{2z^{n+2}-7z^{n+1}+2z^{n}}{(z-1)(z-4)}\right]=2^{n+1}$

Res $\left[U(z)\cdot z^{n-1}\right]_{z=4} == \underset{z\to 4}{Lt}\left[\frac{2z^{n+2}-7z^{n+1}+2z^{n}}{(z-1)(z-2)}\right]=\frac{2\cdot 4^{n+2}-7\cdot 4^{n+1}+2\cdot 4^{n}}{6}=4^{n}$

Hence,

$$Z^{-1}\left[\frac{2z^{3}-7z^{2}+2z}{(z-1)(z-2)(z-4)}\right]=4^{n}+2^{n+1}-1=K_{n}$$

Theorem: 3

$$Z^{-1}\left[\frac{2z^{2}-z}{(z-1)(z-2)}\right]= T_{n} \text{ where } |z|<3$$

Proof:

Let $U(z)=\left[\frac{2z^{2}-z}{(z-1)(z-2)}\right]$

The poles of $U(z)$ are $z=1$ and $z=2$

Now,

Res $\left[U(z)\cdot z^{n-1}\right]_{z=1} = \underset{z\to 1}{Lt}\left[\frac{2z^{n+1}-z^{n}}{(z-2)}\right]=-1$

Res $\left[U(z)\cdot z^{n-1}\right]_{z=2} = \underset{z\to 2}{Lt}\left[\frac{2z^{n+1}-z^{n}}{(z-2)}\right]=3\cdot 2^{n}$

By theorem I, we get

$$Z^{-1}\left[\frac{2z^{2}-z}{(z-1)(z-2)}\right]=3\cdot 2^{n}-1=T_{n}$$

Theorem: 4

$$Z^{-1}\left[\frac{13z^{2}-2z^{3}-14z}{(z-1)(z-2)(z-4)}\right]= Carl_{n}$$

Proof:

Let $U(z)=\left[\frac{2z^{2}-z}{(z-1)(z-2)}\right]$

$$\Rightarrow \frac{U(z)}{z} = \left[\frac{13z - 2z^2 - 14}{(z-1)(z-2)(z-4)}\right] \qquad \text{..........................(1)}$$

By applying partial fraction on the right side of (1), we obtain

$$U(z) = \frac{-z}{(z-1)} - \frac{2z}{(z-2)} + \frac{z}{(z-4)}$$

Taking the inverse Z -Transforms on both sides, we get

$$Z^{-1}[U(z)] = 4^n - 2^{n+1} - 1 = Carl_n$$

Theorem: 5

$$Z^{-1}\left[\frac{z^2}{a(z-a)(z-b)}\right] = \frac{a^n - b^n}{a-b}$$

Proof

$$Z^{-1}\left[\frac{z^2}{a(z-a)(z-b)}\right] = Z^{-1}\left(\frac{z}{a(z-a)} \cdot \frac{z}{(z-b)}\right)$$

We know that

$$Z^{-1}\left[\frac{z}{a(z-a)}\right] = a^{n-1} \ , Z^{-1}\left(\frac{z}{(z-b)}\right) = b^n$$

By theorem (II), we get

$$Z^{-1}\left[\frac{z^2}{a(z-a)(z-b)}\right] = a^{n-1} * b^n = \sum_{m=0}^{n} a^{m-1} \cdot b^{n-m} = b^{n-1} \sum_{m=0}^{n} \left(\frac{a}{b}\right)^{m-1} = \frac{a^n - b^n}{a-b}$$

Remarks 1

If $a = k + \sqrt{k^2+4}$; $b = k - \sqrt{k^2+4}$ then, we observe that

$$Z^{-1}\left[\frac{z^2}{a(z-a)(z-b)}\right] = \frac{\left(k+\sqrt{k^2+4}\right)^n - \left(k-\sqrt{k^2+4}\right)^n}{\left(k+\sqrt{k^2+4}\right) - \left(k-\sqrt{k^2+4}\right)} = k \text{ - Fibonacci sequence.}$$

Remarks 2

When $a = 2$, $b = -1$, we notice that

$$Z^{-1}\left[\frac{z^2}{a(z-a)(z-b)}\right] = \frac{2^n - (-1)^n}{2-(-1)} = \frac{1}{3}\left[2^n - (-1)^n\right] = J_n$$

Theorem: 6

$$Z^{-1}\left[\frac{6z^2 - 6z - z^3}{(z-2)^2(z-1)}\right] = W_n$$

Proof

Let $U(z)=\left[\frac{6z^2-6z-z^3}{(z-2)^2(z-1)}\right]$

$$\Rightarrow \frac{U(z)}{z}=\left[\frac{6z-6-z^2}{(z-2)^2(z-1)}\right] \qquad \text{……………………(2)}$$

By applying partial fraction on the right side of (2), we obtain

$$U(z)=\frac{2z}{(z-2)^2}-\frac{z}{(z-1)}$$

Taking the inverse Z -Transforms on both sides, we get

$$Z^{-1}[U(z)]=n\,2^n-1=W_n$$

Theorem: 7

$$Z^{-1}\left[\frac{z^3-2z^2+2z}{(z-2)^2(z-1)}\right]=C_n$$

Proof

Let $U(z)=\left[\frac{z^3-2z^2+2z}{(z-2)^2(z-1)}\right]$

$$\Rightarrow \frac{U(z)}{z}=\left[\frac{z^2-2z+2}{(z-2)^2(z-1)}\right]$$

Using partial fraction on the right hand side of the above equation, we find that

$$U(z)=\frac{2z}{(z-2)^2}+\frac{z}{(z-1)}$$

Employing the inverse Z -Transforms on both sides, we get

$$Z^{-1}[U(z)]=n\,2^n+1=C_n$$

Conclusion

In this paper, we obtain some special numbers by applying the inverse Z - transforms, the partial fraction method, the residues theorem and convolution theorem. In this manner, one can evaluate some other numbers by applying various properties of some other transforms.

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