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NEW TYPES OF COMPACTNESS

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ABSTRACT

In this paper we developed new concepts in general topology where new compactness spaces different from the known compactness spaces have emerged. We found that an infinite space and β-closed space cannot be strongly compact space. In addition to that each β-closed space that satisfies C_2 property is a semi-compact space. We have also obtained other related results.

INTRODUCTION

The notions of β-open set, β-closed sets play important role in studding many of topological spaces, these notions are introduced and studied by Abd-Almonsef (Abd El-Monsef *et al.,* 1983). Also, they have studied another types of open sets in topological spaces such as semi β-open sets and β-generalized closed set. The concept of g-closed sets in topological spaces was introduced in 1970 by Levine ((Abd El-Monsef *et al.,* 1983), a subset A of (X, τ) is said to be g- closed set, cl(B)⊆U, whenever B⊆U and U is open set. Compactness is one of the most important characteristics that some of the topological spaces may have. The compactness spaces have distinct characteristics lacking in other subsets. Especially compactness provides the possibility to study the space of this property by knowing the number of open sets that cover the space, in order to obtain more specific results wepresent new types of compactness.In this paper, we present a study on the accumulation of topological spaces according to distinct types of open sets. We will focus particularly on β- open sets.

Definitions and basic concepts

In this section we recall some definitions and results, which will be used in this sequel. For detail we refer to e.g. (Abd El-Monsef *et al.,* 1983, 1985, 1986, 1987 and 2005; Dorsett, 1981; Mashhour *et al.,* 1984; Basu *et al.,* 2008; Abd-El-Monsef, 2012). Throughout this paper,

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Definitions 1

- A topological space (X, τ) is called a compact space if every open cover of the space of X such as,$\{T_i$i ∈I} has a finite subcover.
- A topological space (X, τ) is called semi-compact space if every semi-open cover of the space of X such as,$\{T_i$,i∈I} has a finite subcover.
- A topological space (X, τ) is called Strongly compact space if every pre -open cover of the space of X such as $\{T_i; i \in I\}$ has a finite subcover.
- A topological space (X, τ) is called β-compact if every β- open cover of the space of X has a finite subcover.
- Let $f: (X, \tau) \to (Y, \tau^*)$be a function then f is a compact function if the inverse image of each single set element in Y is a compact set in X.
- Let $f: (X, \tau) \to (Y, \tau^*)$be a function then the function f is called M-β continuous function If the inverse image of each β-open set in Y is β-open set of in X.
- A function $f: (X, \tau) \to (Y, \tau^*)$is said to be M-β-open if the direct image of each β-open set in X is a β-open set in Y.

Lemma

Let$f: (X, \tau) \rightarrow (Y, \tau^{*})$be β continuous open function then:
$\forall A \in PO(Y), f^{-1}(A) \in \beta O(X)$
$\forall A \in SO(Y), f^{-1}(A) \in \beta O(X)$

Note

We known that: τ⊆βO(X, τ)
PO(X, τ)⊆βO(X, τ), SO(X, τ)⊆βO(X, τ)

We conclude from this that each β-compact space is a compact and strongly compact space with semi- compact, as shown in the following diagram:

β-compact → strongly compact→compact
↘semi compact↗

The opposite cases are not necessarily correct as the following examples illustrate:

Example

Let X be set a which is not infinite and define the topology of the excluded point T = {T⊂X, x ∉ T} ∪ {X} We found in this space that the family of pre- opensets are:

PO(X)=τ

β-Open sets are : βO(X)=PO(X)

And Space (τ, X) is a compact and strongly compact space, but not β-compact because the family {{x}, x ∈X} is β-open cover for space X but does not contain any finite subcover.

Definitions

Xis said to satisfyC_1 property if each infinite subset of points A⊆X has a non-empty interior.

Lemma
The topological space X satisfiesC_1 property if and only if $X \backslash I_x$ is finite set.

Lemma

The topological space X satisfiesC_1 property if and only if
cl (A) \ A is finite set for each subset of points X.

Lemma

The Space X is strongly compact if and only if it is a compact space and satisfiesC_1 property.

Definition

It is said that Space X satisfies the FCC property if the families ofnon-intersecting and non- empty open sets are finite.

Definition

It is said that Space X satisfies the property C_3 if each infinite subset of points its such as A satisfies$int(cl((A) \neq \emptyset$

Lemma

The topological space X satisfies the property C_3 if and only if cl (T) \ T is finite set for each T ∈ τ.

Lemma

The X space is semi-compact if and only if it satisfies both C_2property and the FCC property.

Definition Connected Space

It is said that the topological space $(X.\tau)$ is a connected space if and only if the two sets X, ∅ are the only two open and closed sets that are one in this space.

Result

X topological space is an disconnected space if and only if two open sets (closed) are not empty, such as A and B, so that both relationships are realized:

$A \cap B = \emptyset , A \cup B = X$

Some results and properties of β-open sets and β-Compact spaces

In this section we will deduce some properties and relations of concepts βsets and β-Compact in a topological space (X,τ) and will introduce some main results.

Definitions

- (E.D.Extremally disconnected space) (Noiri, 1988)

It is said that the topology space (X.τ) is an Extremely disconnected space or E.D if cloture any open set in $(X.\tau)$ is an open set in X which that

- cl (T) ∈ τ, ∀ T ∈τ.
- (Wlanski Space) (Hussein *et al.,* 2012) The space X is said to be Wlanski space (KC) if each is compact set of X is closed set.
- set A is said to be α-open setin a topological space
- (X, τ) if the following relationship is true:
- $A \subseteq \text{int}\left(\text{cl}\left(\text{int}(A)\right)\right)$We will denote the family of α-open sets in this space with the symbol:αO (X).
- A space (X, τ) is called Lindelof if every open cover contains at most a countable subcover.
- A space (X, τ) is called β-Lindelof if every β-open cover contains at most a countable subcover.
- It is said that space (X, τ) is a quasi H-closed space[14]. If from each open cover of space X such as $\{T_i, i \in I\}$ we can write $X = \bigcup_{i=1}^{n} cl(T_i)$, call this short space QHC space.
- A Space (X, τ) is said to be β-closed space [14] if from each of β-open cover For space X like $\{T_i, i \in I\}$ we can write: $X = \bigcup_{i=1}^{n} \beta cl(T_i)$.
- Let A (X, τ) be a topological space, and A is a subset from points of X, it is said that A is β-set if the subspace (A, τ_A) is β-closed space.

Theorem

If (X.τ) is an E. D space, then it is a disconnected space.

Proof

Let T∈τ\\{∅} set how then that: cl(T)∈τ

The universe(X.τ)space E.D on the other handX\cl(T)∈τ

Always have X \ cl (T) ∩cl (T) = ∅ andX \ cl (T) ∪cl (T) = X then the space (X, τ) is a disconnected space.

Note

If (X.τ) is a disconnected space, it is not necessarily an E.D space. This is illustuted by the following example:

Example

Let topological space be (X.τ), X={a,b,c,d}and
τ={X,∅,{b},{c},{d},{b,d},{{c,d},{b,c},{b,c,d},{a,b,c}}
Then that : F={∅,X,{a,c,d},{a,b,d},{a,b,c},{a,c},{a,b},{a,d},{a},{d}}

Note that {d} is an open and closed set in the topological space (X, τ). It is adisconnected.Noting that cl {{b}) = {a, b} where {b} ∈τ, {a, b} ∉τ.We find that (X.τ) is not E.D space.

Lemma

If the topological space (X, τ) is E-D space, then the following relationship is realized:

∀U, V∈τ⇒cl(U∩V)=cl(U)∩cl(V).

Theorem

The topological space (X.τ) is an E.Dspace If and only if the intersection of any two semi-open sets in X is a semi-open set in this space.

Theorem

The topological space (X.τ) is an E.Dspace If and only if is ∀A∈βO(X) we have cl(A)∈τ

Theorem

Let (X.τ) be a topological space which is an E.Dspace and A be a closed set in space X then: int (A) is a closed set in this space.

Lemma

Let X be E.D space so which satisfies the following property: Each β-open set is a semi-closed set, then the next relationship is realized: $\beta O(X) = SO(X) = F = \tau$.

Proof

Let X be an E.D space then for all $A \in \beta O(X)$ we have cl (A) ∈τ and cl (A) ⊆int (cl (A)) and according to the assumption that A is a semi-closed set which that int (cl (A)) ⊆A thus:$A \subseteq cl(A) = int(cl(A)) \subseteq A$ And from it $cl(A) = A \Leftrightarrow$ Aclosed and A = int(A) $\Leftrightarrow A \in \tau$ This means clearly that $A \in \beta O(X)$From the above we conclude that $\beta O(X) = SO(X) = F = \tau$.

Lemma

In any topological space$(X.\tau)$the following statements are equivalent:

- Xis E.D space.
- A∈SO(X,τ) and B∈βO(X,τ)⟶cl(A)∩cl(B)=cl(A∩B).
- A∈SO(X,τ) and B∈βO(X,τ)⟶A∩B∈βO(X,τ).
- βO(X)=SO(X).
- For each A∈PO(X) that βcl(A)∈τ.

Note

In any topology (X, τ) the following containment is satisfies : αO(X)⊆βO(X) and the convers containment is generallyis notsatisfies as the example shows:

Example

Let be X={a,b,c}With topology τ={X,∅,{a},{b,c}} then: βO(X)=P(X) and αO(X)=τ.

Lemma

Let A⊆B⊆X where B is an α -open set in X then :
A∈βO(B)⇔A∈βO(X)

Theorem

If X is a KC space so that each β-closed set is a compactset Then:

1. The Family βO (X) defines a topology on X
2. Space X is an E.DSpace.

Proof

1) We know that $X.\emptyset$ ∈βO(X). In any topological space, in addition to the fact that any union of β-open sets is β-open set, it remains to prove that the intersection of any two β-open sets is β-open set.

Let$A, B \in \beta O(X)$ then each from $A^c, B^c$is β-closed set, therefore, according to the assumption, each is a compact setand thefinite union of a compactsets is a compact set then $A^c \cup B^c$ is compact set In X therefore, and which space X is of KC that $A^c \cup B^c$is a closed set, therefore X \ $A^c \cup B^c$is an open set and thus β-open that X\$(A^c \cup B^c) =$ X\(A ∩ B$)^c =$ A ∩ BThat is, A ∩ B ∈ βO(X)

And from which the family βO (X) defines a topological on X.
2. Let A be β-closed set, therefore, according to the assumption that A is a compact set and B is a semi-closed set, therefore B is β-closed set and according to the hypothesis is acompactset. Let F = A∪B that F be a compactset, because thefinite union a compactsets is a compact set and X is KCspace that F is a Closed set.This means that X \ F is open set any that

X \ F = (X \ A) ∩ (X \ B) ⊆τ⊆βO (X, τ) This means, according to the previous lemma, that X is an E.D spaceWhere X \ B∈SO (X, τ), X \ A∈βO (X, τ).

Theorem

If X isβ-compact space thenX is a finite set.

Proof

Let X be β-compact space and impose an argument that X is an infinite set since that X is β-compact space, so it is strongly compact space, which means that it achieves C_1 and this is equivalent to $I_x \backslash X$is a finite set, and that I_x is (set of points isolated in X) infinite set and This means that $\{\{x\}, x \in I_x\}$families from open sets and it is families infinite and means that X does not achieve the FCC property. This is in contradiction to the fact that the space is semi-compact and therefore the argument is wrong and from it X is a finite set.

Result

The β-compact space is a finite compact space.

Theorem

Let X be an E.Dspace and KC so that each semi-open set in X is a compactset then X is a compact space⇔ X is a semi-compact space.

Proof

From the equivalence we will prove that τ = SO (X) We always have τ⊆SO (X) and any $A \in SO(X)$, that A is a compactset in X, and X is KCspace, then A is closed in X and therefore int (A) is a closed set in X because X is an E.D space. And from it find

$$A \in \tau, int(A) \subseteq A \subseteq cl(int(A)) = int(A)$$

Taking into account the qualitative selection of set A of SO (X), we find that $SO(X) \subseteq \tau$ and henceSO (X) = τ.

Lemma

Each closed subspace in Lindelof space is Lindelofspace.

Result

Each β-Lindelofspace is Lindelof space.

Note

The reversal of the previous result is generally incorrect.

Theorem

Let f: $(X, \tau) \to (Y, \tau^*)$is β-continuous function andsurjective thenif$(X, \tau)$is β-Lindelof space then (Y, τ^*) is of β-Lindelof space.

Proof

Let $\{T_i, i \in I\}$ *be* cover open of space (Y, τ*) then by the fact that function f is a β-continuous function, then $f^{-1}(T_i) \in \beta O(X, \tau), \forall\, i \in I$therefore the family$\{f^{-1}(T_i), i \in I\}$ β-Open cover form of space (X, τ) and β-lindelofspace then,

$$X = \bigcup_{i \in N} f^{-1}(T_i)$$

Taking the direct image of the parties according to f find

$Y =$

This shows that (Y, τ^*) is a Lindelofspace.

Theorem

Let f: $(X, \tau) \to (Y, \tau^*)$be β-closed surjictivefunction such that $f^{-1}(\{y\})$is Lindelofspace in X for each $y \in Y$ then if (Y, τ*) β-LindelofSpace then (X, τ) is a Lindelof Space.

Proof

Let $\{T_i, i \in I\}$be cover open to space X and be $y \in Y$ point of how then $f^{-1}(\{y\})$ is Lindelof space in X, so there is a subcover that can be counteble from any open cover, and according to$\{T_i, i \in I\}$ is a cover for space X, it forms an open cover for the set $f^{-1}(\{y\})$and from it $f^{-1}(\{y\}) \subseteq \bigcup_{i \in N} T_i$. And put $T_y = \bigcup_{i \in N} T_i$ *and* $G_y = Y \backslash f(X \backslash T_y)$ *then* $X \backslash T_y$is a closed set in X.Thus f(X\$T_y$)is β-closed set in Y.

Which that : $\{G_y^a, a \in \Lambda, y \in Y\}$form β-open cover for space Y, By taking the inverse image of $G_y$set according to f we find

$$f^{-1}(G_y) = f^{-1}\left(Y \backslash f(X \backslash T_y)\right) = f^{-1}(Y) \backslash f^{-1}\left(f(X \backslash T_y)\right) = X \backslash f^{-1}\left(f(X \backslash T_y)\right)$$

That is, $f^{-1}(G_y) \subseteq T_y$ is for each y∈Y and Y is β -Lindelofspace then Y = $\bigcup_{i \in N} G_y^a$ and from it$f^{-1}(Y) = \bigcup_{a \in N} f^{=1}(G_y^a) = \bigcup_{a \in N} T_y^a = \bigcup_{i \in N}(\bigcup_{i \in N} T_i^{\,a})$

Which that : X = $\bigcup_{i \in N} T_i$and thus X is a Lindelofspace.

Note

The result of the definition and the fact that each of the following two relationships βcl (A) ⊆cl (A) and τ⊆βO (X) investigate in any topological space that each β-closed space is QHC space, and the inverse state is not necessarily true as the following example shows:

Theorem

Assuming X be QHCspace andβ^*-regular E.D space then X is β-closed space.

Proof

Let $\{T_i, i \in I\}$be cover how from β-open sets to space X then$\{cl(T_i), i \in I\}$ is open cover him and X is QHCspace, then X = $\bigcup_{i=1}^{n} cl(T_i)$and X is a β^*-regular space then cl$(T_i) = \beta cl_\theta(T_i)$ for each$1 \le i \le n$ and βcl$(T_i) = \beta cl_\theta(T_i)$ according to theoretically (15.3) We conclude that $X = \bigcup_{i=1}^{n} \beta cl(T_i)$ his shows that X is β −closed space.

Note

Any infinite space that satisfies C_1property can not be β-closed space.

Proof

Let X be infinite space and satisfiesC_1, property and impose a argument that X is β-closed then $X = \bigcup_{i=1}^{n} \beta cl(T_i)$ whatever $\{T_i, i \in I\}$be open cover from β −open sets and from it**:**

$$X = \bigcup_{i=1}^{n}((\beta cl(T_i)\backslash T_i)\bigcup T_i) = \bigcup_{i=1}^{n}(\beta cl(T_i)\backslash T_i)\bigcup(\bigcup_{i=1}^{n} T_i)$$

And X space satisfies C_1property then $\beta cl(T_i)\backslash T_i$ finiteset whatever $i \in I$ and we know that finite union of the finite sets is set finite any set $A = \bigcup_{i=1}^{n}(\beta cl(T_i)\backslash T_i)$a finite set is β-compact which is $X = A \cup (\bigcup_{i=1}^{n} T_i$

This means that X isβ-compact space, which means X is finite set. This is contrary to the fact that X is an infinite space. The space is not β-closed.

Result

Each infinite and β-closed space can not be astrongly compact space. This follows from the fact that each strongly compact space is compact space with C_1 property.

Note

It is not necessarily true that the β-closed space is a semi-compact space.

Theorem

Each β-closed space and satisfies C_2 property is a semi-compact space.

Proof

Let $\{T_i, i \in I\}$ is cover semi-open how to space X therefore

β-open cover and X is β-closed space then $X = \bigcup_{i=1}^{n} \beta cl(T_i)$ and βcl(A) ⊆ cl(A); ∀ A ⊆ Xthen $X = \bigcup_{i=1}^{n} cl(T_i)$, Noting that no matter what A semi- open setthen:

$cl(int(A)) \subseteq cl(A) \subseteq cl(int(A))$ and thus: A = cl(int(A))

From which we find: $\beta cl(T_i)\backslash T_i \subseteq cl(T_i)\backslash T_i = cl(int(T_i))\backslash T_i \subseteq cl(int(T_i))\backslash Int\,(T_i)$

Whichever is 1 ≤ i ≤ n,and the space satisfiesC_2 property then theset $cl(int((T_i))\backslash int(T_i)$ is finite and thus $\beta cl(T_i)\backslash T_i$ is finite and from it :$X = \bigcup_{i=1}^{n}((\beta cl T_i)\backslash T_i) \cup T_i) = \bigcup_{i=1}^{n}(\beta cl(T_i)\backslash T_i) \cup (\bigcup_{i=1}^{n} T_i)$

We know that thefinite union set is a finiteset, which that the set $A = \bigcup_{i=1}^{n}(\beta cl(T_i)\backslash T_i)$ finiteset is semi a compact and from it$X = A \cup (\bigcup_{i=1}^{n} T_i) = \bigcup_{\alpha=1}^{m} T_\alpha$which that X is a semi-compact space.

Note

Not all semi-compact space is β-closed space.

Theorem

If X is a semi-compact space and E.D then it isβ-closed space.

Proof

Let $\{ T_i, i \in I\}$ be semi-open cover of X that X is E.Dspace that the family { βcl(T_i); $i \in I$} forms open cover and therefore semi open of space, and according to semi-compact space that $X = \bigcup_{i=1}^{n} \beta cl(T_i)$ and this means that X is β −closed set.

Note

Not all a compactspace should have β-closed space.

Result

From the previous theorem, according to the fact that each space is a semi-compact space, is compactspace that each compactspace and an E.Dspace is a β-closed space.

Note

It is not necessary that each β-closed space is a compact space.

Theorem

if X is β-closed topological space, it is determined that τ = F Then X is a compact space.

Proof

Let $\{ T_i, i \in I\}$ be cover open how of space X, so that space X is β-closed space, then $X = \bigcup_{i=1}^{n} \beta cl(T_i)$ And from which $X = \bigcup_{i=1}^{n}((\beta cl(T_i\backslash T_i) \cup T_i)$. It is known that each closed set is β -closed generalized set thus βcl(T_i)$\backslash T_i$does not contain β-closed set other than the empty set according to the lemma (3.2) Thus the only open set that covers βcl(T_i)$\backslash T_i$ is X. This means that it is a compact set, 1 ≤ i ≤ n From this we find that: $X = \bigcup_{i=1}^{n} T_i$ thus X is a compact space.

Note

It is not necessarily true that β-closed setis β-closed space.

Theorem

Let $B \subseteq$ Xwhere B is α-open set and let be A ⊆ B, Then that:
A is β-set in X ⇔ A is β-set in B.

Proof

Let the family $\{T_i, i \in I\}$be cover how to β-open sets For set A in B therefore $A \subseteq \bigcup_{i \in I} T_i$ This is equivalent to:$T_i \in \beta O(X), \forall i \in I$

Thus the family $\{\{ T_i, i \in I\}$ be form β-open cover for set A in X but A is β-set in X by force Thus $A \subseteq \bigcup_{i=1}^{n} \beta cl(T_i)$ is thus A β-set in B. On the inverse :, let the family $\{T_i, i \in I\}$ are cover how from β-open sets of set A in X then the family $\{T_i, i \in I\}$ are β-Open cover of set A in B ;B being α-open set in X. And A is β-set in B, $A \subseteq \bigcup_{i=1}^{n} \beta cl(T_i)$.This means that A is a β-set in X.

Theorem

Let X be topological space and B is β-set in X and Let T be β-closed setsuch that T⊆BThen B \ T is β-set in X.

Proof

Let $\{T_i, i \in I\}$be open cover of β-open sets for the set B \ T then the family$\{T_i, i \in I\} \cup \{T\}$ forms of β-Open cover for set B and B be β-set thatB $\subseteq (\bigcup_{i=1}^{n} \beta cl(T_i)) \cup \beta cl(T)$ which that $B = (B \backslash T) \cup T \subseteq (\bigcup_{i=1}^{n} \beta cl(T_i)) \cup T$ Hence, B $\subseteq \bigcup_{i=1}^{n} \beta cl(T_i)$ thereforeB \ Tis β-set in X.

Theorem

If X is topological space and F is β-regular set and A is β-set then A ∩ F is a β-set in X.

Proof

Let $\{T_i, i \in I\}$be open cover of the β-open sets for set A∩F in X is that A ∩ F $\subseteq \bigcup_{i \in I} T_i$ therefore $A = (A \cap F) \cup (A \backslash F) \subseteq (\bigcup_{i \in I} T_i) \cup (X \backslash F)$

But F is β-regularsetthen the family$\{T_i, i \in I\} \cup \{X \backslash F\}$ is β-open cover for set A but A isβ -set thus implying:

$$A \subseteq \left(\bigcup_{i=1}^{n} \beta cl(T_i\right) \cup (\beta cl(X \backslash F)) = (\bigcup_{i=1}^{n} \beta cl(T_i)) \cup (X \backslash F)$$

Hence, A ∩ F $\subseteq \bigcup_{i=1}^{n} \beta cl(T_i)$ hence A ∩ F is a β-set in

Theorem

Let X be a compact topological space and E.D space then each closed set is a β-set in X.

Proof

Let $\{T_i, i \in I\}$be β-open cover how ofset A in X

then the family $\{T_i, i \in I\} \cup \{X \backslash A\}$ is β-open cover For space X and X is an E.D space.

The family $\{\beta cl\ (T_i), i \in I\} \cup \{X \backslash A\}$ form an open cover to it and according to the universe of X is compacspace t, that:

$$X = (\bigcup_{i=1}^{n} \beta cl\ (T_i)) \cup (\beta cl(X \backslash A)) = (\bigcup_{i=1}^{n} \beta cl\ (T_i)) \cup (X \backslash A)$$

And from it: A $\subseteq \bigcup_{i=1}^{n} T_i$.

Theorem

Each compactset in E.D space is β-closed subspace.

Proof

Let $\{T_i, i \in I\}$beβ-open cover How of set A in X.
Then the family $\{\beta cl(T_i), i \in I\}$ form an open cover of set Aaccording to X is an E.DspaceandA is a compact set, then A $\subseteq \bigcup_{i=1}^{n} \beta cl(T_i)$.So $(A, \tau_A))$ is β-closed subspace.

Conclusion and recommendation

Inthis paper wedevelopment new concepts in general topology where new compactness spaces different from the known compactness spaces have emerged.We found that an infinite space and β-closed space can not be strongly compact space and each β-closed space that satisfies C_2 property is a semi-compact space and each semi-compact space is β-closed space when it is E.Dspace. Also if X is β^*regular topological space and A∈ βO (X) so that it is β-set then that A is a compactset in X. We recommend following up this study according to the generalized species of this type of open sets and studying the connection of the topological spaces based on β-open sets in particular.

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