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Research Article

INTEGRAL SOLUTIONS OF HOMOGENEOUS BI-QUADRATIC EQUATION WITH FIVE UNKNOWNS $2\left(x^2-y^2\right)\left(2\left(x^2+y^2\right)-3xy\right)=11\left(z^2-w^2\right)T^2$

Gopalan, M.A., *Usha Rani, T.R. and Vidhyalakshmi, S.

Department of Mathematics, Shrimathi Indira GandhiCollege, Trichy, 620002, India

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ABSTRACT

We obtain infinitely many non-zero integer quintuples satisfying the Biquadratic equation with five unknowns .Various interesting properties among the values of x, y, z,w andT are presented.

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INTRODUCTION

The theory of Diophantine equations offers a rich variety of fascinating problems. In particular, biquadratic diophantine equations, homogeneous and non-homogeneous have aroused the interest of numerous mathematicians since antiquity (Dickson, 1952, Mordell, 1969, Carmichae, 1959, Telang, 1996, Nigel and Smart 1999). In this context, one may refer (Gopalan *et al* 2009, 2010, a, b, c, 2012, Meena *et al* 2014) for various problems on the biquadratic diophantine equations with four variables and (Gopalan *et al* 2009, 2011, 2014, Vidhyalakshmi *et al* 2014.) for 5 variables However, often we come across non-homogeneous biquadratic equations and as such one may require its integral solution in its most general form. It is towards this end, this paper concerns with the problem of determining non-trivial integral solutions of the homogeneous equation with five unknowns given by

$$2\left(x^2-y^2\right)\left(2\left(x^2+y^2\right)-3xy\right)=11\left(z^2-w^2\right)T^2$$

A few relations among the solutions are presented.

MATERIALS AND METHODS

The homogeneous bi-quadratic equation under consideration is

$$2\left(x^2-y^2\right)\left(2\left(x^2+y^2\right)-3xy\right)=11\left(z^2-w^2\right)T^2 \quad \text{..........} (1)$$

Introducing the linear transformations

$$x=u+v, y=u-v, z=2u+v, w=2u-v \quad \text{..........} (2)$$

Corresponding author: *Usha Rani, T.R.,
Department of Mathematics, Shrimathi Indira GandhiCollege, Trichy, 620002, India.

in (1) it is written as

$$u^2 + 7v^2 = 11T^2 \quad \text{(3)}$$

$$T = a^2 + 7b^2 \quad \text{(4)}$$

Write 11 as

$$11 = (2 + i\sqrt{7})(2 - i\sqrt{7}) \quad \text{(5)}$$

Using (4), (5) in (3) and using method of factorization, define

$$u + i\sqrt{7}v = (2 + i\sqrt{7})(a + i\sqrt{7}b)^2$$

Equating real and imaginary parts in the above equation, we get

$$u = 2a^2 - 14b^2 - 14ab$$
$$v = a^2 - 7b^2 + 4ab$$

Substituting the values of u,v in (2) we get

$$\left.\begin{array}{l} x = 3a^2 - 21b^2 - 10ab \\ y = a^2 - 7b^2 - 18ab \\ z = 5a^2 - 35b^2 - 24ab \\ w = 3a^2 - 21b^2 - 32ab \end{array}\right\} \quad \text{(6)}$$

Thus (4) and (6) represent the non-zero distinct integer solutions to (1)

Properties

1) $x(a+1, a^2) + 21t_{4,a^2} - t_{8,a} + 20P_a^5 \equiv 3 \pmod 8$

2) $x(a+1, a) - x(a, a+1) - 24Gno_{a+1} = 0$

3) $z(1, b) - y(1, b) + t_{58,b} \equiv 4 \pmod{33}$

4) $3T(a, 2a^2+1) + x(a, 2a^2+1) + 30OH_a$ is a nasty number.

5)Each of the following is a perfect square:

$i)\, x(1, b) + y(1, b) + 28PR_b$

$ii)\, 2(x(a, 2a^2+1) + y(a, 2a^2+1) - w(a, 2a^2+1) + T(a, 2a^2+1) - 12OH_a$ $iii)\, 2(y(a, a+1) + T(a, a+1) + 36t_{3,a}$

One may also write (3) as

$$u^2 + 7v^2 = 11T^2 * 1 \quad (7)$$

Write 1 as

$$1 = \frac{(3 + i4\sqrt{7})(3 - i4\sqrt{7})}{121} \quad (8)$$

Using (4),(5) and (8) in (7) and employing the method of factorization, define

$$u + i\sqrt{7}v = \frac{(2 + i\sqrt{7})(3 + i4\sqrt{7})(a + i\sqrt{7}b)^2}{11}$$

Equating real and imaginary parts in the above equation and performing some algebra,
the non-zero integral solutions to (1) are given by

$$\left.\begin{aligned} x &= -a^2 + 7b^2 - 18ab \\ y &= -3a^2 + 21b^2 - 10ab \\ z &= -3a^2 + 21b^2 - 32ab \\ w &= -5a^2 + 35b^2 - 24ab \\ T &= a^2 + 7b^2 \end{aligned}\right\}$$

Properties

1) $y(a,11a) - 3x(a,11a)$ is a perfect square

2) $x(a+1,a) + 2t_{8,a} + 18\Pr_a \equiv -1(\mathrm{mod}\,2)$

3) $y(a+1,a^2) - z(a+1,a^2) + w(a+1,a^2) + 5T(a+1,a^2) + 4P_a^5$ is abiquadratic integer

4) $5T(a,9a) - w(a,9a) - 10t_{4,a}$ is a cubical integer

5) $x(a,2a^2-1) + 18SO_a - 28t_{4,a^2} + 2t_{31,a} \equiv 7(\mathrm{mod}\,27)$

However, it is worth mentioning that the integers 11 and 1 in (7) may also be represented as follows:

$$\left.\begin{aligned} 11 &= \frac{(1 + i5\sqrt{7})(1 - i5\sqrt{7})}{16} \\ 1 &= \frac{(1 + i3\sqrt{7})(1 - i3\sqrt{7})}{64} \end{aligned}\right\} \quad (9)$$

Making use of the above representations (9) and employing (2), four more patterns of integer solutions to (1) are obtained and they are as follows:

Pattern1

$$\left.\begin{aligned} x &= 6A^2 - 42B^2 - 68AB \\ y &= -4A^2 + 28B^2 - 72AB \\ z &= 7A^2 - 49B^2 - 138AB \\ w &= -3A^2 + 21B^2 - 142AB \\ T &= 4(A^2 + 7B^2) \end{aligned}\right\}$$

Pattern2

$$\left.\begin{aligned} x &= 11(-118P^2 + 826Q^2 - 540PQ) \\ y &= 11(-156P^2 + 1092Q^2 + 8PQ) \\ z &= 11(-255P^2 + 1785Q^2 - 806PQ) \\ w &= 11(-293P^2 + 2051Q^2 - 258PQ) \\ T &= 4*11^2(P^2 + 7Q^2) \end{aligned}\right\}$$

Pattern3

$$\left.\begin{aligned} x &= -24A^2 + 168B^2 - 272AB \\ y &= -52A^2 + 364B^2 - 120AB \\ z &= -62A^2 + 434B^2 - 468AB \\ w &= -90A^2 + 630B^2 - 316AB \\ T &= 4^2(A^2 + 7B^2) \end{aligned}\right\}$$

Pattern4

$$\left.\begin{aligned} x &= -192P^2 + 1344Q^2 - 640PQ \\ y &= -224P^2 + 1568Q^2 + 192PQ \\ z &= -400P^2 + 2800Q^2 - 864PQ \\ w &= -432P^2 + 3024Q^2 - 32PQ \\ T &= 4^3(P^2 + 7Q^2) \end{aligned}\right\}$$

Conclusion

In (2), the representations for z and w may be considered as z=2uv+1, w=2uv-1 and thus, one may be getting six more choices of integer solutions to (1).As bi-quadratic equations with five unknowns are rich in variety one may search for other patterns of solutions along with their properties.

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