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Research Article

COMMON RANDOM FIXED POINTS OF RANDOM MULTI-VALUED OPERATOR ON COMPLETE METRIC SPACES

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ABSTRACT

In this paper, we using Kannan type and Chatterjea contractions and obtained some random fixed point results for multi-valued contractive conditions in the complete metric spaces. Our results generalize and improve some main results in the literature and references therein.

INTRODUCTION

Random fixed point theory is playing an increasing role in mathematics and applied sciences. At present, it received considerable attention due to enormous application in many important areas such as nonlinear analysis, probability theory and the study of random equations arising in various applied areas. Random fixed point theorems for random contraction mappings on separable complete metric spaces were first proved by Spacek [Spajcek, 1955] and Hans (Hans, 1961, Hans, 1957). The survey article by Bharucha-Reid [Bharucha-Reid, 1976] in 1976 attracted the attention of several mathematician and gave wings to this theory. Itoh [Itoh, 1979] extended Spacek's result and Hans's theorem to multivalued contraction mappings. In an attempt to construct iterations for finding fixed points of random operators defined on linear spaces. This iteration and some other random iterations based on the same ideas have been applied for finding solutions of random operators.

Definitions

Throughout this paper, let (Ω, Σ) be a measurable space and X an arbitrary metric space. We denote by 2^X, and $B(X)$ the family of all nonempty subset of X, the duality space of X, the family of all nonempty subset of X . Throughout this paper, let (Ω, Σ) be a measurable space and X is a metric space with metric d. Let 2^X denote a collection of all nonempty subsets of X, $B(X)$ a collection of all nonempty closed subsets of X, and H the Hausdorff metric on $B(X)$, i.e.,

$H(A, B) = \max\{\sup_{\xi(\omega)\in A} d(\xi(\omega), B), \quad \sup_{\eta(\omega),\in B} d(\eta(\omega), A)\}$, $A, B \in B(X)$.

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An element $\xi(\omega) \in X$ is said to be a fixed point of a multivalued map $T : \Omega \times X \to X \to 2^X$ if $\xi(\omega) \to T(\omega, \xi(\omega))$. A multivalued map $T: \Omega \times X \to B(X)$ is said to be a contraction if for a fixed constant $h \in (0, 1)$ and for each $\omega \in \Omega$,

$H(T(\omega, \xi(\omega)), T(\omega, \eta(\omega))) \leq hd(\xi(\omega), \eta(\omega))$.

Definition 2.1. A mapping $\xi: \Omega \to X$ is said to be measurable if for each $B \in$

$B(X)$, $\{\omega : \xi(\omega) \in B\} \in \Sigma$.

Definition 2.2. A mapping $T: \Omega \times X \to X$ is called a random operator if for each

$\omega \in \Omega$, $T(\omega, \xi(\omega)) = \xi(\omega)$ is measurable.

Definition 2.3. A multi-valued mapping $T: \Omega \to 2^X$ is said to be measurable if

for any $B \in B(X)$, $T^{-1}(B) = \{\omega \in \Omega : T(\omega) \cap B = \varphi\} \in \Sigma$.

Definition 2.4. A mapping $T: \Omega \to 2^X$ is called a random multi-valued mapping

if for each $\omega \in \Omega$, $T(., \omega) : \Omega \to 2^X$ *is measurable.*

A mapping T of X into itself is called a contraction if there exists a positive real number $\alpha < 1$ with the property

$$d(Tx, Ty) \leq \alpha d(x, y) \tag{2.1}$$

for all $x, y \in X$. On the other hand Kanan [Kannan, 1969] proved that If T is self mapping of a complete metric space X into itself satisfying:

$$d(Tx, Ty) \leq \eta[d(x, Tx) + d(y, Ty)]$$

for all $x, y \in X$; where $\eta \in [0, 1/2]$. Then T has unique fixed point in X. Moreover, Fisher [Hans, 1957] proved the result with

$$d(Tx, Ty) \leq \mu[d(x, Tx) + d(y, Ty)] + \delta d(x, y)$$

for all $x, y \in X$; where $\mu, \delta \in [0, 1/2]$. Then T has unique fixed point in X. A similar conclusion was also obtained by Chaterjee [Chatterjee, 1972] proved the result with

$$d(Tx, Ty) \leq \mu[d(x, Ty) + d(y, Tx)]$$

for all $x, y \in X$; where $\mu \in [0, 1/2]$. Then T has unique fixed point in X.

RESULTS

Theorem 3.1. Let X be a complete metric space. Let $T: \Omega \times X \to CB(X)$ be a continuous random multivalued operator. If there exist measurable mappings $\alpha(\omega)$, $\beta(\omega)$, $\gamma(\omega)$, $\delta(\omega)$ in (0, 1) with $\alpha(\omega) + \beta(\omega) + \gamma(\omega) + \delta(\omega) < 1$ satisfying following condition:

$$\begin{aligned} H(T(\omega,x),T(\omega,y)) \leq \alpha d(x,y) + \beta \frac{d(x,T(\omega,x)) \cdot d(y,T(\omega,y))}{d(x,y)} \\ + \gamma \frac{d(x,T(\omega,x)) \cdot d(x,T(\omega,y)) + d(y,T(\omega,y)) \cdot d(y,T(\omega,x))}{d(x,T(\omega,y)) + d(y,T(\omega,x))}, \end{aligned} \tag{3.1}$$

For each $\omega \in \Omega$, *for all distinct* $x, y \in X$. *Then T has a fixed point.*

Proof. Let $xi_{n+1} T x_n$, then there exists $x_{n+2} \in T x_{n+1}$ an arbitrary measurable mapping and choose a measurable mapping $\xi_1 : \Omega \to X$ such that $\xi_0(\omega) \in T(\omega, \xi_1(\omega))$ for each $\omega \in \Omega$. In this way we define a sequence $\{\xi_n(\omega)\}$ for each $\omega \in \Omega$ then from

(3.1), we have

$$\begin{aligned}
&d(\xi_n(\omega),\xi_{n+1}(\omega)) = H(T\xi_n(\omega),T\xi_{n+1}(\omega))\\
&\leq \alpha d(\xi_n(\omega),\xi_{n+1}(\omega)) + \beta\frac{d(\xi_n(\omega),T\xi_n(\omega))\cdot d(\xi_{n+1}(\omega),T\xi_{n+1}(\omega))}{d(\xi_n(\omega),\xi_{n+1}(\omega))}\\
&\quad +\gamma\frac{d(\xi_n(\omega),T\xi_n(\omega))\cdot d(\xi_n(\omega),T\xi_{n+1}(\omega)) + d(\xi_{n+1}(\omega),T\xi_{n+1}(\omega))\cdot d(\xi_{n+1}(\omega),T\xi_n(\omega))}{d(\xi_n(\omega),T\xi_{n+1}(\omega)) + d(\xi_{n+1}(\omega),T\xi_n(\omega))}\\
&= \alpha d(\xi_n(\omega),\xi_{n+1}(\omega)) + \beta\frac{d(\xi_n(\omega),\xi_{n+1}(\omega))\cdot d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}{d(\xi_n(\omega),\xi_{n+1}(\omega))}\\
&\quad +\gamma\frac{d(\xi_n(\omega),\xi_{n+1}(\omega))\cdot d(\xi_n(\omega),\xi_{n+2}(\omega)) + d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))\cdot d(\xi_{n+1}(\omega),\xi_{n+1}(\omega))}{d(\xi_n(\omega),\xi_{n+2}(\omega)) + d(\xi_{n+1}(\omega),\xi_{n+1}(\omega))}\\
&\leq \alpha d(\xi_n(\omega),\xi_{n+1}(\omega)) + \beta d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) + \gamma d(\xi_n(\omega),\xi_{n+1}(\omega))\\
&\leq (\alpha+\gamma)d(\xi_n(\omega),\xi_{n+1}(\omega)) + \beta d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))\\
&\leq \frac{(\alpha+\gamma)}{(1-\beta)}d(\xi_n(\omega),\xi_{n+1}(\omega)),
\end{aligned}$$

which implies that

$$d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) \leq k(\xi_n(\omega),\xi_{n+1}(\omega)), \tag{3.2}$$

where $k = \frac{(\alpha+\gamma)}{(1-\beta)} < 1$.
Similarly, if $\xi_{n+2}(\omega) \in T\xi_{n+1}(\omega) \subset B$ and $\xi_{n+3}(\omega) \in T\xi_{n+2}(\omega) \subset A$, using (??),we get

$$\begin{aligned}
&d(\xi_{n+2}(\omega),\xi_{n+3}(\omega)) \leq H(T\xi_{n+1}(\omega),T\xi_{n+2}(\omega))\\
&\leq \alpha d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) + \beta\frac{d(\xi_{n+1}(\omega),T\xi_{n+2}(\omega))\cdot d(\xi_{n+2}(\omega),T\xi_{n+2}(\omega))}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}\\
&\quad +\gamma\frac{d(\xi_{n+1}(\omega),T\xi_{n+2}(\omega))\cdot d(\xi_{n+1}(\omega),T\xi_{n+2}(\omega)) + d(\xi_{n+2}(\omega),T\xi_{n+2}(\omega))\cdot d(\xi_{n+2}(\omega),T\xi_{n+1}(\omega))}{d(\xi_{n+1}(\omega),T\xi_{n+2}(\omega)) + d(\xi_{n+2}(\omega),T\xi_{n+1}(\omega))}\\
&= \alpha d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) + \beta\frac{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))\cdot d(\xi_{n+2}(\omega),\xi_{n+2}(\omega))}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}\\
&\quad +\gamma\frac{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))\cdot d(\xi_{n+1}(\omega),\xi_{n+3}(\omega)) + d(\xi_{n+2}(\omega),\xi_{n+3}(\omega))\cdot d(\xi_{n+2}(\omega),\xi_{n+2}(\omega))}{d(\xi_{n+1}(\omega),\xi_{n+3}(\omega)) + d(\xi_{n+2}(\omega),\xi_{n+2}(\omega))}\\
&\leq \alpha d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) + \beta d(\xi_{n+2}(\omega),\xi_{n+3}(\omega)) + \gamma d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))\\
&\leq (\alpha+\gamma)d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) + \beta d(\xi_{n+2}(\omega),\xi_{n+3}(\omega))\\
&\leq \frac{(\alpha+\gamma)}{(1-\beta)}d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))\\
&\leq kd(\xi_{n+1}(\omega),\xi_{n+2}(\omega)).
\end{aligned} \tag{3.3}$$

Now, from (3.2) and (3.3), we have

$$\begin{aligned}
d(\xi_{n+2}(\omega),\xi_{n+3}(\omega)) &\leq kd(\xi_{n+1}(\omega),\xi_{n+2}(\omega))\\
&\leq k[k(\xi_n(\omega),\xi_{n+1}(\omega))\\
&\leq k^2(\xi_n(\omega),\xi_{n+1}(\omega)).
\end{aligned}$$

Using induction, we obtain a sequence $\xi(\omega) \in T(\omega,\xi_{2n-1}(\omega)) \subset B$, $\xi_{2n+1}(\omega) \in T\xi(\omega) \subset A$, and

$$d(\xi(\omega), \xi_{2n+1}(\omega)) \leq H(T(\omega, \xi_{2n-1}(\omega)), T(\omega, \xi(\omega))$$
$$\leq kd(x_{2n-1}, \xi(\omega))$$
$$d(\xi(\omega), \xi_{2n+1}(\omega)) \leq k^{2n} d(\xi_n(\omega), \xi_{n+1}(\omega)).$$

$$d(\xi_{2n+1}(\omega), \xi_{2n+2}(\omega)) \leq k^{2n+1} d(\xi_n(\omega), \xi_{n+1}(\omega)).$$

In general, we get

$$d(x_n, x_{n+1}) \leq k^n d(\xi_n(\omega), \xi_{n+1}(\omega)).$$

Thus $\lim_{n\to\infty} d(x_n, x_{n+1}) = 0$ as $n \to \infty$. This implies that $\lim_{n\to\infty} d(\xi(\omega), \xi_{2n+1}(\omega)) = 0$ and $\lim_{n\to\infty} d(\xi_{2n+2}(\omega), \xi_{2n+1}(\omega)) = 0$. Therefore both sequences $\{\xi_{2n}\}$ and $\{\xi_{2n+1}\}$ are bounded, then the $\{\xi_{2n}\}$ and $\{\xi_{2n+1}\}$ have subsequences converging to some element $\xi(\omega)$. Furthermore

$$0 \leq d(\xi(\omega), \xi_{2n_k-1})$$
$$\leq d(\xi(\omega), \xi_{2n_k}) + d(\xi_{2n_k}, \xi_{2n_k-1})$$
$$\leq d(\xi(\omega), \xi_{2n_k}).$$

Therefore $d\xi(\omega), \xi_{2n_k-1}) \to 0$. Since $d(0 \leq d(\xi_{2n}, T(\omega, \xi(\omega)) \leq d(\xi_{2n-1}, \xi(\omega))$, we have $d(\xi(\omega), T(\omega, \xi(\omega)) = 0$, hence $\xi(\omega)$ is a random fixed point of T. This completes the proof.

Theorem 3.2. Let X be a complete metric space. Let $T: \Omega \times X \to CB(X)$ be a continuous random multivalued operator. If there exist measurable mappings α_1, α_2, α_3, β_1, β_2, β_3 and β_4 in (0, 1) such that

$$\begin{aligned} H(T(\omega, x), T(\omega, y)) \geq & \frac{\alpha_1(\omega)\Big(d(x, T(\omega, x))d(y, T(\omega, y))\Big)}{d(x, y)} \\ & + \frac{\alpha_(\omega)\Big[1 + d(x, T(\omega, x))\Big]d(x, T(\omega, y))}{d(x, y)} \\ & + \frac{\alpha_3(\omega)\Big(d(y, T(\omega, y))d(x, T(\omega, y))\Big)}{d(x, y)} \\ & + \beta_1(\omega)d(x, T(\omega, x)) + \beta_2(\omega)d(y, T(\omega, y)) \\ & + \beta_3(\omega)d(x, T(\omega, y)) + \beta_4(\omega)d(y, T(\omega, x)), \end{aligned}$$

(3.4)

for all distinct $x, y \in X$, $\omega\Omega$, where α_1, α_2, α_3, β_1, β_2, β_3 and $\beta_4 \in R^+$, $\alpha_1(\omega) + \beta_1(\omega) + \beta_2(\omega) + \beta_4(\omega) > 0$. Then T has a fixed point.

Proof. Let ξ_0: $\Omega \to X$ be an arbitrary measurable mapping and choose a mea- surable mapping $\xi_1 : \Omega \to X$ such that $\xi_0(\omega) \in T(\omega, \xi_1(\omega))$ for each $\omega \in \Omega$. In this way we define a sequence $\{\xi_n(\omega)\}$ for each $\omega \in \Omega$ as follows:

$$\xi_n(\omega) \in T(\omega, \xi_{n+1}(\omega)) for n = 1, 2, \ldots \tag{3.5}$$

Now consider

$$
\begin{aligned}
d(\xi_n(\omega),\xi_{n+1}(\omega)) &= H(T(\omega,\xi_{n+1}(\omega)),T(\omega,\xi_{n+2}(\omega)))\\
&\geq \frac{\alpha_1(\omega)\Big(d(\xi_{n+1}(\omega),T(\omega,\xi_{n+1}(\omega)))d(\xi_{n+2}(\omega),T(\omega,\xi_{n+2}(\omega)))\Big)}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}\\
&\quad+\frac{\alpha_(\omega)\Big[1+d(\xi_{n+1}(\omega),T(\omega,\xi_{n+1}(\omega)))\Big]d(\xi_{n+1}(\omega),T(\omega,\xi_{n+2}(\omega)))}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}\\
&\quad+\frac{\alpha_3(\omega)\Big(d(\xi_{n+2}(\omega),T(\omega,\xi_{n+2}(\omega)))d(\xi_{n+1}(\omega),T(\omega,\xi_{n+2}(\omega)))\Big)}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}\\
&\quad+\beta_1(\omega)d(\xi_{n+1}(\omega),T(\omega,\xi_{n+1}(\omega)))+\beta_2(\omega)d(\xi_{n+2}(\omega),T(\omega,\xi_{n+2}(\omega)))\\
&\quad+\beta_3(\omega)d(\xi_{n+1}(\omega),T(\omega,\xi_{n+2}(\omega)))+\beta_4(\omega)d(\xi_{n+2}(\omega),T(\omega,\xi_{n+1}(\omega)))\\
&= \frac{\alpha_1(\omega)\Big(d(\xi_{n+1}(\omega),\xi_n(\omega)))d(\xi_{n+2}(\omega),\xi_{n+1}(\omega)))\Big)}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}\\
&\quad+\frac{\alpha_(\omega)\Big[1+d(\xi_{n+1}(\omega),\xi_n(\omega)))\Big]d(\xi_{n+1}(\omega),\xi_{n+1}(\omega)))}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}\\
&\quad+\frac{\alpha_3(\omega)\Big(d(\xi_{n+2}(\omega),T(\omega,\xi_{n+2}(\omega)))d(\xi_{n+1}(\omega),\xi_{n+1}(\omega)))\Big)}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}\\
&\quad+\beta_1(\omega)d(\xi_{n+1}(\omega),\xi_n(\omega)))+\beta_2(\omega)d(\xi_{n+2}(\omega),T(\omega,\xi_{n+2}(\omega)))\\
&\quad+\beta_3(\omega)d(\xi_{n+1}(\omega),T(\omega,\xi_{n+2}(\omega)))+\beta_4(\omega)d(\xi_{n+2}(\omega),T(\omega,\xi_{n+1}(\omega)))\\
&= \frac{\alpha_1(\omega)\Big(d(\xi_{n+1}(\omega),\xi_n(\omega)))d(\xi_{n+2}(\omega),\xi_{n+1}(\omega)))\Big)}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}\\
&\quad+\frac{\alpha_(\omega)\Big[1+d(\xi_{n+1}(\omega),\xi_n(\omega)))\Big]d(\xi_{n+1}(\omega),\xi_{n+1}(\omega)))}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}\\
&\quad+\frac{\alpha_3(\omega)\Big(d(\xi_{n+2}(\omega),T(\omega,\xi_{n+2}(\omega)))d(\xi_{n+1}(\omega),\xi_{n+1}(\omega)))\Big)}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}\\
&\quad+\beta_1(\omega)d(\xi_{n+1}(\omega),\xi_n(\omega)))+\beta_2(\omega)d(\xi_{n+2}(\omega),T(\omega,\xi_{n+2}(\omega)))\\
&\quad+\beta_3(\omega)d(\xi_{n+1}(\omega),T(\omega,\xi_{n+2}(\omega)))\\
&\quad++\beta_4(\omega)d(\xi_{n+2}(\omega),\xi_{n+1}(\omega))+\beta_4(\omega)d(\xi_{n+1}(\omega),\xi_n(\omega))\\
&\geq (\alpha_1(\omega)+\beta_1(\omega)+\beta_4(\omega))d(\xi_n(\omega),\xi_{n+1}(\omega))\\
&\quad+(\beta_2(\omega)+\beta_4(\omega))d(\xi_{n+2}(\omega),\xi_n(\omega))\\
&\Rightarrow d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))\\
&\leq \frac{1-(\alpha_1(\omega)+\beta_1(\omega)+\beta_4(\omega))}{\beta_2(\omega)+\beta_4(\omega)}[d(\xi_{n+2}(\omega),\xi_n(\omega))\\
&\quad+d(\xi_{n+1}(\omega),\xi_n(\omega))]\xi_{n+1}(\omega))\\
&\Rightarrow d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))\leq k(\omega)d(\xi_n(\omega),\xi_{n+1}(\omega)),
\end{aligned}
$$

where $k = k(\omega) = \frac{1-(\alpha_1(\omega)+\beta_1(\omega)+\beta_4(\omega))}{\beta_2(\omega)+\beta_4(\omega)}$. So, in general

$$d(\xi_n(\omega), \xi_{n+1}(\omega)) \leq kd(\xi_{n-1}(\omega), \xi_n(\omega)) forn = 1, 2, 3, dots$$
$$\Rightarrow d(\xi_n(\omega), \xi_{n+1}(\omega)) \leq k^n d(\xi_0(\omega), \xi_1(\omega))$$

which implies that

$$d(\xi_n(\omega), \xi_{n+1}(\omega)) \leq k^n d(\xi_0(\omega), \xi_1(\omega)) \tag{3.6}$$

Now we shall prove that for each $\omega \in \Omega$, $\{\xi_n(\omega)\}$ is a Cauchy sequence. For this for every positive integer p, we have

$$\begin{aligned} d(\xi_n(\omega), \xi_{n+p}(\omega)) &\leq d(\xi_n(\omega), \xi_{n+1}(\omega)) + d(\xi_{n+1}(\omega), \xi_{n+12}(\omega)) + \cdots + d(\xi_{n+p-1}(\omega), \xi_{n+p}(\omega)) \\ &\leq (k^n + k^{n+1} + k^{n+2} + \cdots + k^{n+p-1} d(\xi_0(\omega), \xi_1(\omega)) \\ &= k^n(1 + k + k^2 + \cdots + k^{p-1} d(\xi_0(\omega), \xi_1(\omega)) \\ &< \frac{k^n}{(1-k)} d(\xi_0(\omega), \xi_1(\omega)) \end{aligned}$$

which tends to zero as $n \to \infty$. It follows that $\{\xi_n(\omega)\}$ is a Cauchy sequence and there exists a measurable mapping $\xi : \Omega : X$ such that $\xi_n(\omega) \in \xi(\omega)$ for each $\omega \in \Omega$.

Existence of random fixed point: Since T is a surjective self map, so there exist a function $g : \Omega \to X\ X$ such that

$$\xi(\omega) \in T(\omega, \xi(\omega)). \tag{3.7}$$

Now

$$\begin{aligned} d(\xi_n(\omega), \xi(\omega)) &= H(\xi_n(\omega), T(\omega, g(\omega))) \\ &\geq \frac{\alpha_1(\omega)\Big(d(\xi_{n+1}(\omega), \xi_n(\omega))d(g(\omega), T(\omega, g(\omega)))\Big)}{d(\xi_{n+1}(\omega), g(\omega))} \\ &\quad + \frac{\alpha_(\omega)\Big[1 + d(\xi_{n+1}(\omega), \xi_n(\omega))\Big]d(\xi_{n+1}(\omega), T(\omega, g(\omega)))}{d(\xi_{n+1}(\omega), g(\omega))} \\ &\quad + \frac{\alpha_3(\omega)\Big(d(g(\omega), T(\omega, g(\omega)))d(\xi_{n+1}(\omega), T(\omega, g(\omega)))\Big)}{d(\xi_{n+1}(\omega), g(\omega))} \\ &\quad + \beta_1(\omega)d(\xi_{n+1}(\omega), \xi_n(\omega)) + \beta_2(\omega)d(g(\omega), T(\omega, g(\omega))) \\ &\quad + \beta_3(\omega)d(\xi_{n+1}(\omega), T(\omega, g(\omega))) + \beta_4(\omega)d(g(\omega), T(\omega, \xi_{n+1}(\omega)) \\ &= \frac{\alpha_1(\omega)\Big(d(\xi_{n+1}(\omega), \xi_n(\omega)))d(g(\omega), \xi_{n+1}(\omega)))\Big)}{d(\xi_{n+1}(\omega), g(\omega))} \\ &+ \frac{\alpha_(\omega)\Big[1 + d(\xi_{n+1}(\omega), \xi_n(\omega)))\Big]d(\xi_{n+1}(\omega), \xi_{n+1}(\omega)))}{d(\xi_{n+1}(\omega), g(\omega))} \\ &+ \frac{\alpha_3(\omega)\Big(d(g(\omega), T(\omega, g(\omega)))d(\xi_{n+1}(\omega), \xi_{n+1}(\omega)))\Big)}{d(\xi_{n+1}(\omega), g(\omega))} \\ &+ \beta_1(\omega)d(\xi_{n+1}(\omega), \xi_n(\omega))) + \beta_2(\omega)d(g(\omega), T(\omega, g(\omega))) \\ &+ \beta_3(\omega)d(\xi_{n+1}(\omega), T(\omega, g(\omega))) + \beta_4(\omega)d(g(\omega), \xi_n(\omega)). \end{aligned}$$

Since $\{\xi_{n+1}(\omega)\}$ is a subsequence of $\{\xi_n(\omega)\}$, so $\{\xi_n(\omega)\} \to \{\xi(\omega)\} \Rightarrow \{\xi_{n+1}(\omega)\} \to \{\xi(\omega)\}$, when $n \to \infty$.

$$\begin{aligned}
0 \geq\ & \frac{\alpha_1(\omega)\Big(d(\xi(\omega),\xi(\omega)))d(g(\omega),\xi(\omega)))\Big)}{d(\xi(\omega),g(\omega))} \\
&+ \frac{\alpha_(\omega)\Big[1+d(\xi(\omega),\xi(\omega)))\Big]d(\xi(\omega),\xi(\omega)))}{d(\xi(\omega),g(\omega))} \\
&+ \frac{\alpha_3(\omega)\Big(d(g(\omega),g(\omega))d(\xi(\omega),\xi(\omega)))\Big)}{d(\xi(\omega),g(\omega))} \\
&+ \beta_1(\omega)d(\xi(\omega),\xi(\omega))) + \beta_2(\omega)d(g(\omega),g(\omega)) \\
&+ \beta_3(\omega)d(\xi(\omega),g(\omega)) + \beta_4(\omega)d(g(\omega),\xi(\omega)) \\
0 \geq\ & (\beta_2(\omega)+\beta_4(\omega))d(g(\omega),\xi(\omega)) \Rightarrow d(\xi(\omega),g(\omega)) = 0,
\end{aligned}$$

as $\beta_2(\omega) + \beta_4(\omega) > 0$. It follows that

$$\xi(\omega) = g(\omega) \tag{3.8}$$

The fact (3.8) along with (3.7) shows that $\xi(\omega)$. This completes the proof.

Theorem 3.3. Let X be a complete metric spaces. Let $T: \Omega \times X \rightarrow CB(X)$ be a continuous random multivalued operator

$$\begin{aligned}
H(T(\omega,x),T(\omega,y)) \geq q\max\Big\{& d(x,y), \frac{\Big(d(x,T(\omega,x))d(y,T(\omega,y))\Big)}{d(x,y)}, \\
& \frac{\Big[1+d(x,T(\omega,x))\Big]d(x,T(\omega,y))}{d(x,y)}, \\
& \frac{\Big(d(y,T(\omega,y))d(x,T(\omega,y))\Big)}{d(x,y)}\Big\},
\end{aligned} \tag{3.9}$$

for each $x, y \in X$, $\omega\Omega$ and $q > 1$. Here H represents the Hausdroff metric on $CB(X)$ induced by the metric d. Then T has a fixed point.

Proof. Let a sequence $\xi_n(\omega)$ as in proof of theorem 3.2. We claim that the inequality (3.9) for $x = \xi_{n+1}(\omega)$ and $y = \xi_{n+2}(\omega)$ we have

$$\begin{aligned}
H(T(\omega,\xi_{n+1}(\omega)),T(\omega,\xi_{n+2}(\omega))) \geq\ & q\max\Big\{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)), \\
& \frac{\Big(d(\xi_{n+1}(\omega),T(\omega,\xi_{n+1}(\omega))d(\xi_{n+2}(\omega),T(\omega,\xi_{n+2}(\omega))\Big)}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}, \\
& \frac{\Big[1+d(\xi_{n+1}(\omega),T(\omega,\xi_{n+1}(\omega))\Big]d(\xi_{n+1}(\omega),T(\omega,\xi_{n+2}(\omega))}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}, \\
& \frac{\Big(d(\xi_{n+2}(\omega),T(\omega,\xi_{n+2}(\omega))d(\xi_{n+1}(\omega),T(\omega,\xi_{n+2}(\omega))\Big)}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}\Big\}, \\
=\ & q\max\Big\{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)), \\
& \frac{\Big(d(\xi_{n+1}(\omega),\xi_n(\omega))d(\xi_{n+2}(\omega),\xi_{n+1}(\omega))\Big)}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}, \\
& \frac{\Big[1+d(\xi_{n+1}(\omega),\xi_n(\omega))\Big]d(\xi_{n+1}(\omega),\xi_{n+1}(\omega))}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}, \\
& \frac{\Big(d(\xi_{n+2}(\omega),\xi_{n+1}(\omega))d(\xi_{n+1}(\omega),\xi_{n+1}(\omega))\Big)}{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))}\Big\} \\
=\ & q\max\Big\{d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)), d(\xi_{n+1}(\omega),\xi_n(\omega))\Big\},
\end{aligned}$$

Case I. If $d(\xi_n(\omega)\xi_{n+1}(\omega)) \geq qd(\xi_n(\omega), \xi_{n+1}(\omega)) \Rightarrow 1 \geq q$,which is contradiction.
Case II. If $d(\xi_n(\omega)\xi_{n+1}(\omega)) \geq qd(\xi_{n+1}(\omega), \xi_{n+2}(\omega)) \Rightarrow d(\xi_{n+1}(\omega), \xi_{n+2}(\omega)) \leq \frac{1}{q} d(\xi_n(\omega)\xi_{n+1}(\omega)) \Rightarrow$ $d(\xi_{n+1}(\omega), \xi_{n+2}(\omega)) \leq kd(\xi_n(\omega)\xi_{n+1}(\omega))$, where $k = \frac{1}{q} < 1$, since $q > 1$. So, in gen-eral

$$d(\xi_n(\omega)\xi_{n+1}(\omega)) \le kd(\xi_{n-1}(\omega)\xi_n(\omega)),$$

for all $n = 1, 2, 3,$ dots.

$$d(\xi_n(\omega)\xi_{n+1}(\omega)) \le k^n d(\xi_0(\omega)\xi_1(\omega)), \tag{3.10}$$

We can prove that for each $\omega \in \Omega$, so $\{\xi_n(\omega)\}$ is a Cauchy sequence using (3.10) as proved in theorem 3.3 and since X is a complete space, so there exists a measurable mapping $\xi : \Omega \to X$ such that $\{\xi_n(\omega)\} \to \xi(\omega)$ for each $\omega \in \Omega$.

Existence of random fixed point: Since T is a surjective self map, so there exist a function g: $\Omega \to X\ X$ such that

$$\xi(\omega) \in T(\omega, \xi(\omega)). \tag{3.11}$$

Now

$$\begin{aligned} d(\xi_n(\omega), \xi(\omega)) = H(T(\omega, \xi_{n+1}(\omega), T(\omega, g(\omega))) \\ \ge q \max \Big\{ d(\xi_{n+1}(\omega), \xi_{n+2}(\omega)), \\ \frac{\big(d(\xi_{n+1}(\omega), T(\omega, \xi_{n+1}(\omega))d(\xi_{n+2}(\omega), T(\omega, g(\omega))\big)}{d(\xi_{n+1}(\omega), \xi_{n+2}(\omega))}, \\ \frac{\big[1 + d(\xi_{n+1}(\omega), T(\omega, \xi_{n+1}(\omega))\big] d(\xi_{n+1}(\omega), T(\omega, g(\omega))}{d(\xi_{n+1}(\omega), \xi_{n+2}(\omega))}, \\ \frac{\big(d(\xi_{n+2}(\omega), T(\omega, \xi_{n+2}(\omega))d(\xi_{n+1}(\omega), T(\omega, g(\omega))\big)}{d(\xi_{n+1}(\omega), \xi_{n+2}(\omega))} \Big\}, \\ = q \max \Big\{ d(\xi_{n+1}(\omega), \xi_{n+2}(\omega)), \\ \frac{\big(d(\xi_{n+1}(\omega), \xi_n(\omega))d(\xi_{n+2}(\omega), \xi_{n+1}(\omega))\big)}{d(\xi_{n+1}(\omega), \xi_{n+2}(\omega))}, \\ \frac{\big[1 + d(\xi_{n+1}(\omega), \xi_n(\omega))\big] d(\xi_{n+1}(\omega), \xi_{n+1}(\omega))}{d(\xi_{n+1}(\omega), \xi_{n+2}(\omega))}, \\ \frac{\big(d(\xi_{n+2}(\omega), \xi_{n+1}(\omega))d(\xi_{n+1}(\omega), \xi_{n+1}(\omega))\big)}{d(\xi_{n+1}(\omega), \xi_{n+2}(\omega))} \Big\}. \end{aligned}$$

Since $\{\xi_{n+1}(\omega)\}$ is a subsequence of $\{\xi_n(\omega)\}$, so $\{\xi_n(\omega)\} \to \{\xi(\omega)\} \Rightarrow \{\xi_{n+1}(\omega)\} \to \{\xi(\omega)\}$, when $n \to \infty$.

$$\begin{aligned} d(\xi_n(\omega), \xi(\omega)) \ge q \max \Big\{ d(\xi(\omega), \xi(\omega)), \\ \frac{\big(d(\xi(\omega), \xi_n(\omega))d(\xi(\omega), \xi(\omega))\big)}{d(\xi(\omega), \xi(\omega))}, \\ \frac{\big[1 + d(\xi(\omega), \xi_n(\omega))\big] d(\xi(\omega), \xi(\omega))}{d(\xi(\omega), \xi(\omega))}, \\ \frac{\big(d(\xi(\omega), \xi(\omega))d(\xi(\omega), \xi(\omega))\big)}{d(\xi(\omega), \xi(\omega))} \Big\}. \end{aligned}$$

Now, we conclude that

$$0 \ge q \max\{d(\xi(\omega), g(\omega))\} \Rightarrow d(\xi(\omega), g(\omega)) = 0 as q > 1 \Rightarrow, \xi(\omega) = g(\omega). \tag{3.12}$$

The fact (3.12) along with (3.11) shows that $\xi(\omega)$ is a fixed point of T. This completes the proof.

Theorem 3.4. Let X be a complete metric spaces. Let T: $\Omega \times X \to CB(X)$ be a continuous random multivalued operator. If there exist measurable mappings

a, b, c: $\Omega \in (0, 1)$ such that

$$d^2(\omega,x),T(\omega,y)) \geq a(\omega)d(x,T(\omega,x))d(x,y)+b(\omega)d(y,T(\omega,y))d(x,y) \\ +c(\omega)d(x,T(\omega,x))d(y,T(\omega,y)) \tag{3.13}$$

for each $x, y \in X$, ωΩ, where a, b, c, $\in \mathbb{R}^+$, $b(\omega) > 0$ and $a(\omega) + b(\omega) + c(\omega) > 1$metric. Then T has a fixed point.

Proof. Let a sequence $\xi_n(\omega)$ as in proof of theorem 3.2. We claim that the inequality (3.13) for $x = \xi_{n+1}(\omega)$ and $y = \xi_{n+2}(\omega)$ we have

$$\begin{aligned} d^2(\omega,\xi_{n+1}(\omega)),T(\omega,\xi_{n+2}(\omega))) &\geq a(\omega)d(\xi_{n+1}(\omega),T(\omega,\xi_{n+1}(\omega)))d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) \\ &\quad +b(\omega)d(\xi_{n+2}(\omega),T(\omega,\xi_{n+2}(\omega)))d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) \\ &\quad +c(\omega)d(\xi_{n+1}(\omega),T(\omega,\xi_{n+1}(\omega)))d(\xi_{n+2}(\omega),T(\omega,\xi_{n+2}(\omega))) \\ &= a(\omega)d(\xi_{n+1}(\omega),\xi_n(\omega))d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) \\ &\quad +b(\omega)d(\xi_{n+2}(\omega),\xi_{n+2}(\omega))d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) \\ &\quad +c(\omega)d(\xi_{n+1}(\omega),\xi_n(\omega))d(\xi_{n+2}(\omega),\xi_{n+2}(\omega)) \\ &\geq (a(\omega)+b(\omega)+c(\omega))d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) \\ &\quad \min\{d(\xi_n(\omega),\xi_{n+1}(\omega)),d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))\}. \end{aligned}$$

Case I. If

$$\begin{aligned} d^2(\xi_n(\omega),\xi_{n+1}(\omega)) &\geq (a(\omega)+b(\omega)+c(\omega))d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))d(\xi_n(\omega),\xi_{n+1}(\omega)) \\ &\Rightarrow d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) \\ &\leq \frac{1}{(a(\omega)+b(\omega)+c(\omega))}d(\xi_n(\omega),\xi_{n+1}(\omega)) \\ &\Rightarrow d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) \\ &\leq k_1 d(\xi_n(\omega),\xi_{n+1}(\omega)), \end{aligned}$$

where $k = \frac{1}{(a(\omega)+b(\omega)+c(\omega))} < 1$ as $(a(\omega)+b(\omega)+c(\omega)) > 1$.

Case II. If

$$\begin{aligned} d^2(\xi_n(\omega),\xi_{n+1}(\omega)) &\geq (a(\omega)+b(\omega)+c(\omega))d(\xi_{n+1}(\omega),\xi_{n+2}(\omega))d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) \\ &\Rightarrow d^2(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) \\ &\leq \frac{1}{(a(\omega)+b(\omega)+c(\omega))}d^2(\xi_n(\omega),\xi_{n+1}(\omega)) \\ &\Rightarrow d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) \\ &\leq \left[\frac{1}{(a(\omega)+b(\omega)+c(\omega))}\right]^{\frac{1}{2}}d(\xi_n(\omega),\xi_{n+1}(\omega)) \\ &\Rightarrow d(\xi_{n+1}(\omega),\xi_{n+2}(\omega)) \\ &\leq k_2(\omega)d(\xi_n(\omega),\xi_{n+1}(\omega)), \end{aligned}$$

where $k_2(\omega) = \left[\frac{1}{(a(\omega)+b(\omega)+c(\omega))}\right]^{\frac{1}{2}} < 1$ as $(a(\omega)+b(\omega)+c(\omega)) > 1.$ Assume that $k_2(\omega) = k(\omega) = \max\{k_1(\omega), k_2(\omega)\}$, then $k < 1.$ Hence, in inductively, we have

$$d(\xi_n(\omega),\xi_{n+1}(\omega)) \leq d(\xi_{n-1}(\omega),\xi_n(\omega)) for n = 1,2,3,dots.$$

It follows that

$$d(\xi_n(\omega),\xi_{n+1}(\omega)) \leq k^n d(\xi_0(\omega),\xi_1(\omega)) \tag{3.14}$$

We can prove that for each $\omega \in \Omega$, so $\{\xi_n(\omega)\}$ is a Cauchy sequence using (3.14) as proved in theorem 3.2. Now since X is a complete space, so $\{\xi_n(\omega)\} \to \xi(\omega)$.

Existence of random fixed point: Since T is a surjective self map, hence there exist a function g: $\Omega \to X X$ such that

$$\xi(\omega) \in T(\omega,\xi(\omega)). \tag{3.15}$$

We conclude that from (3.13), we obtain

$$\begin{aligned} d(\xi_n(\omega), \xi(\omega)) &= H(T(\omega, \xi_{n+1}(\omega)), T(\omega, g(\omega)) \\ &\geq a(\omega)d(\xi_{n+1}(\omega), T(\omega, \xi_{n+1}(\omega)))d(\xi_{n+1}(\omega), g(\omega)) \\ &\quad + b(\omega)d(g(\omega), T(\omega, g(\omega)))d(\xi_{n+1}(\omega), g(\omega)) \\ &\quad + c(\omega)d(\xi_{n+1}(\omega), T(\omega, \xi_{n+1}(\omega)))d(g(\omega), T(\omega, g(\omega))) \\ &= a(\omega)d(\xi_{n+1}(\omega), (\omega, \xi_{n+1}(\omega))d(\xi_{n+1}(\omega), g(\omega)) \\ &\quad + b(\omega)d(g(\omega), \Xi(\omega)))d(\xi_{n+1}(\omega), g(\omega)) \\ &\quad + c(\omega)d(\xi_{n+1}(\omega), (\omega, \xi_{n+1}(\omega)))d(g(\omega), (\omega, g(\omega))) \end{aligned}$$

Since $\{\xi_{n+1}(\omega)\}$ is a subsequence of $\{\xi_n(\omega)\}$, so $\{\xi_n(\omega)\} \rightarrow \{\xi(\omega)\} \Rightarrow \{\xi_{n+1}(\omega)\} \rightarrow \{\xi(\omega)\}$, when $n \rightarrow \infty$. We have

$$\begin{aligned} d(\xi(\omega), \xi(\omega)) \leq\ & a(\omega)d(\xi(\omega), (\omega, \xi(\omega))d(\xi(\omega), g(\omega)) \\ &+ b(\omega)d(g(\omega), \Xi(\omega)))d(\xi(\omega), g(\omega)) \\ &+ c(\omega)d(\xi(\omega), (\omega, \xi(\omega)))d(g(\omega), (\omega, g(\omega))) \end{aligned}$$

$$0 \geq b(\omega)d^2(\xi(\omega), g(\omega)) \Rightarrow d(\xi(\omega), g(\omega)) = 0 as b(\omega) > 0 \Rightarrow \xi(\omega) = g(\omega). \quad (3.16)$$

The fact (3.16) along with (3.15) shows that $\xi(\omega)$ is a fixed point of T. This completes the proof.

REFERENCES

Bharucha-Reid, A.T. 1976. Fixed point theorems in probabilistic analysis. *Bull. Am. Math. Soc.* 82(5), 641-657.

Chatterjee, S.K. 1972. Fixed point theorems. Comptes. *Rend. Acad. Bulgaria Sci.*,Vol.25, 727-730.

Fisher, B. 1976. A fixed point theorem for compact metric space Publ. *Inst Math,* Vol.25. 193-194.

Hans, O. 1957. Reduzierende zufllige transformationen. Czechoslov. Math. J. 7(82), 154-158.

Hans, O. 1961. Random operator equations. In: Proceedings of 4th Berkeley Sympos. *Math. Statist. Prob.,* vol. II, part I, pp. 185-202. University of California Press, Berkeley.

Itoh, S. 1979. Random fixed-point theorems with an application to random differential equations in Banach spaces. *J. Math. Anal. Appl.*, 67(2), 261-273.

Kannan, R. 1969. Some results on fxed points-II .Bulletin of Calcutta *Math.Soc.,*Vol.60.71-76.

Spajcek, A. 1955. Zufallige Gleichungen. Czechoslov. *Math. J.* 5(80), 462-466.

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