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Research Article

ON THE NON-HOMOGENEOUS CUBIC EQUATION WITH FIVE UNKNOWNS $9(x^3-y^3)=z^3-w^3+12p^2+16$

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ABSTRACT

An attempt has been made to determine all possible integer solutions satisfying the non-homogeneous cubic equation with five unknowns given by $9(x^3-y^3)=z^3-w^3+12p^2+16.$

INTRODUCTION

It is well known that cubic Diophantine equations [homogeneous & non-homogeneous] are rich in variety[1-3] and one may refer [4-19] for the non- zero distinct integer solutions satisfying the respective equation. It is to be noted that the integer solutions of the equations presented in the above references are determined by applying the method of factorization and employing the method of cross multiplication. Also, for some cases, the linear transformations have been introduced to obtained the integer solutions for the corresponding equation. It is worth mentioning that, it may be possible to obtain all integer solutions through a single approach which is the main thrust of this paper. In this context, an attempt has been made to obtain all possible integer solutions to the non-homogeneous cubic equation with five unknowns given by

$$9(x^3-y^3)=z^3-w^3+12p^2+16.$$

Method of Analysis

The non-homogeneous cubic equation with five unknowns under consideration is

$$9(x^3-y^3)=z^3-w^3+12p^2+16 \qquad \text{……………………………………………………………. (1)}$$

Introducing the linear transformations

$$x=u+1,\ y=u-1,\ z=v+1,\ w=v-1 \qquad \text{……………………………………………………………. (2)}$$

in (1), it is written as

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$$9u^2=v^2+2p^2 \quad \text{……………(3)}$$

Dividing the above equation by u^2 and writing

$$X=\frac{v}{u}, \qquad \& Y=\frac{p}{u} \quad \text{……………(4)}$$

We have,

$$X^2+2Y^2=9 \quad \text{……………(5)}$$

Where X, Y are rational.

Note that, $X=1, Y=2$ satisfies (5) and consider the relation between X & Y as

$$Y=t(x-1)+2 \quad \text{……………(6)}$$

where 't' is rational.

Substituting (6) in (5) and simplifying the resulting equation is

$$X^2[2t^2+1]+2X[4t-2t^2]+2t^2-8t-1=0 \quad \text{……………(7)}$$

Treating (7) as a quadratic in X and solving for X, we have

$$X=1,\frac{2t^2-6t-1}{2t^2+1} \quad \text{……………(8)}$$

From (6) & (8), we have

$$Y=2,\frac{-2t^2-2t+2}{2t^2+1} \quad \text{……………(9)}$$

The values X=1, Y= 2 lead to the integer solutions of (1) to be

$$x=u+1, y=u-1, z=u+1, w=u-1, p=2u$$

Considering the rational values of X and Y in (8) & (9) and using (2), we have

$$x=u+1$$
$$y=u-1$$
$$z=\frac{u(2t^2-6t-1)}{2t^2+1}+1$$
$$w=\frac{u(2t^2-6t-1)}{2t^2+1}-1$$
$$p=\frac{u(-2t^2-2t+2)}{2t^2+1}$$

Choosing u as

$$u=\alpha(2t^2+1), \alpha\in Z-\{0\}$$

The integer solutions of (1) are given by

$$\left.\begin{aligned} x&=\alpha(2t^2+1)+1\\ y&=\alpha(2t^2+1)-1\\ z&=\alpha(2t^2-6t-1)+1\\ w&=\alpha(2t^2-6t-1)-1\\ p&=\alpha(2-2t-2t^2)\end{aligned}\right\} \quad \text{……………(10)}$$

A few numerical illustrations are given below

T	α	x	y	Z	W	p
$\frac{1}{2}$	2	3	2	-6	-8	1
$\frac{1}{3}$	9	12	10	-24	-26	10
$\frac{2}{3}$	9	18	16	-36	-38	-2
$\frac{1}{4}$	8	10	8	-18	-20	11
$\frac{1}{5}$	25	28	26	-52	-54	38
$\frac{1}{7}$	49	52	50	-88	-90	82

Remark

Replacing t by $\frac{1}{T}$ and α by αT^2 in the solutions (10), another set of integer solutions to (1) are as follows:

$$\left.\begin{aligned} x &= \alpha(T^2+2)+1 \\ y &= \alpha(T^2+2)-1 \\ z &= \alpha(2-T^2-8T)+1 \\ w &= \alpha(2-T^2-8T)-1 \\ p &= \alpha(2T^2-2T-4) \end{aligned}\right\} \qquad \ldots\ldots (11)$$

It is seen that (10) & (11) represent all possible integer solutions to (1).

Conclusion

In this paper, an attempt has been made to present all possible integer solutions for the cubic equation with five unknowns given in the title. To conclude one may search for other choices of multi degree equations with multi variables for which the above method is applicable.

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