IJIRR

International Journal of Information Research and Review
Vol. 03, Issue, 11, pp. 3058-3065, November, 2016

Research Article

A NEW CLASS OF GENERALIZED BIPOLAR VAGUE SETS

Cicily Flora, S. and Arockiarani, I.

Department of Mathematics, Nirmala College for Women, Coimbatore-18

ARTICLE INFO

Article History:

Received 28th August, 2016
Received in revised form
22nd September, 2016
Accepted 24th October, 2016
Published online November, 30th 2016

Keywords:

Bipolar Vague Topology,
Bipolar Vague Continuity,
Bipolar Vague Generalized Closed Set.

ABSTRACT

The focus of this paper is to formulate a new class of generalized bipolar vague closed sets in bipolar vague topological spaces and confered its properties.

INTRODUCTION

Generalized closed sets in general topology were studied by Levine (Levine, 1970). After the introduction of the concept of fuzzy sets by Zadeh (Zadeh, 1965) several researches were conducted on the generalizations of the notion of the fuzzy set. In the traditional fuzzy sets, the membership degree of element range over the interval [0, 1]. There are several kinds of fuzzy set extensions in the fuzzy set theory, for example, intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets etc. Bipolar-valued fuzzy sets, which was introduced by Lee(9, 10) is an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [-1, 1]. The notion of vague set theory introduced by Gau W.L and Buehrer D.J (Gau, 1993), as a generalization of Zadeh's fuzzy set.

The objective of this paper is to introduce the concept of bipolar vague generalized closed set.

2. PRELIMINARIES

Definition 2.1(Lee, 2000)**:** Let X be the universe. Then a bipolar valued fuzzy sets, A on X is defined by positive membership function μ_A^+, that is $\mu_A^+ : X \to [0,1]$, and a negative membership function μ_A^-, that is $\mu_A^- : X \to [-1,0]$. For the sake of simplicity, we shall use the symbol $A = \{\langle x, \mu_A^+(x), \mu_A^-(x)\rangle : x \in X\}$.

Definition 2.2(Lee, 2000)**:** Let A and B be two bipolar valued fuzzy sets then their union, intersection and complement are defined as follows:

$(i)\ \mu_{A\cup B}^+(x) = \max\{\mu_A^+(x), \mu_B^+(x)\}$.

$(ii)\ \mu_{A\cup B}^-(x) = \min\{\mu_A^-(x), \mu_B^-(x)\}$

$(iii)\ \mu_{A\cap B}^+(x) = \min\{\mu_A^+(x), \mu_B^+(x)\}$

Corresponding author: Cicily Flora, S.
Department of Mathematics, Nirmala College for Women, Coimbatore-18.

$(iv)\, \mu^-_{A\cap B}(x) = \max\{\mu^-_A(x), \mu^-_B(x)\}$

$(v)\, \mu^+_{\bar{A}}(x) = 1 - \mu^+_A(x)$ *and* $\mu^-_{\bar{A}}(x) = -1 - \mu^-_A(x)$ for all $x \in X$.

Definition 2.3(7): A vague set A in the universe of discourse U is a pair (t_A, f_A) where $t_A : U \to [0,1]$, $f_A : U \to [0,1]$ are the mapping such that $t_A + f_A \le 1$ for all $u \in U$ .The function t_A and f_A are called true membership function and false membership function respectively. The interval $[t_A, 1-f_A]$ is called the vague value of u in A, and denoted by $v_A(u)$, i.e $v_A(u) = [t_A(u), 1-f(u)]$.

Definition 2.4(Gau, 1993**):** Let A be a non-empty set and the vague set A and B in the form $A = \{\langle x, t_A(x), 1-f_A(x)\rangle : x \in X\}$, $B = \{\langle x, t_B(x), 1-f_B(x)\rangle : x \in X\}$.

Then

(i) $A \subseteq B$ if and only if $t_A(x) \le t_B(x)$ and $1 - f_A(x) \le 1 - f_B(x)$

$(ii)\, A \cup B = \{< \max(t_A(x), t_B(x)), \max(1-f_A(x), 1-f_B(x)) > / x \in X\}$.

$(ii)\, A \cap B = \{< \min(t_A(x), t_B(x)), \min(1-f_A(x), 1-f_B(x)) > / x \in X\}$.

(iv) $\overline{A} = \{\langle x, f_A(x), 1-t_A(x)\rangle : x \in X\}$.

Definition 2.5(Arockiarani, 2016**):** Let X be the universe of discourse. A bipolar-valued vague set A in X is an object having the form $A = \{\langle x, [t^+_A(x), 1-f^+_A(x)], [-1-f^-_A(x), t^-_A(x)]\rangle : x \in X\}$ where $[t^+_A, 1-f^+_A] : X \to [0,1]$ and $[-1-f^-_A, t^-_A] : X \to [-1,0]$ are the mapping such that $t^+_A + f^+_A \le 1$ and $-1 \le t^-_A + f^-_A$. The positive membership degree $[t^+_A(x), 1-f^+_A(x)]$ denotes the satisfaction region of an element x to the property corresponding to a bipolar-valued set A and the negative membership degree $[-1-f^-_A(x), t^-_A(x)]$ denotes the satisfaction region of x to some implicit counter property of A. For a sake of simplicity, we shall use the notion of bipolar vague set $v^+_A = [t^+_A, 1-f^+_A]$ and $v^-_A = [-1-f^-_A, t^-_A]$.

Definition 2.6(4): A bipolar vague set $A = [v^+_A, v^-_A]$ of a set U with $v^+_A = 0$ implies that $t^+_A = 0, 1-f^+_A = 0$ and $v^-_A = 0$ implies that $t^-_A = 0, -1-f^-_A = 0$ for all $x \in U$ is called zero bipolar vague set and it is denoted by 0.

Definition 2.7(4): A bipolar vague set $A = [v^+_A, v^-_A]$ of a set U with $v^+_A = 1$ implies that $t^+_A = 1, 1-f^+_A = 1$ and $v^-_A = -1$ implies that $t^-_A = -1, -1-f^-_A = -1$ for all $x \in U$ is called unit bipolar vague set and it is denoted by 1.

3. BIPOLAR VAGUE TOPOLOGICAL SPACE

Definition 3.1: Let $A =< x, [t^+_A, 1-f^+_A][-1-f^-_A, t^-_A] >$ and $B =< x, [t^+_B, 1-f^+_B][-1-f^-_B, t^-_B] >$ be two bipolar vague sets then their union, intersection and complement are defined as follows:

1. $A \cup B = \{(x, [t^+_{A\cup B}(x), 1-f^+_{A\cup B}(x)], [-1-f^-_{A\cup B}(x), t^-_{A\cup B}(x)]) / x \in X\}$ where

$t^+_{A\cup B}(x) = \max\{t^+_A(x), t^+_B(x)\}$, $t^-_{A\cup B}(x) = \min\{t^-_A(x), t^-_B(x)\}$ and

$1 - f^+_{A\cup B}(x) = \max\{1-f^+_A(x), 1-f^+_B(x)\}$,

$-1 - f^-_{A\cup B}(x) = \min\{-1-f^-_A(x), -1-f^-_B(x)\}$.

2. $A \cap B = \{(x, [t^+_{A\cap B}(x), 1-f^+_{A\cap B}(x)], [-1-f^-_{A\cap B}(x), t^-_{A\cap B}(x)]) / x \in X\}$ where

$t^+_{A\cap B}(x) = \min\{t^+_A(x), t^+_B(x)\}$, $t^-_{A\cap B}(x) = \max\{t^-_A(x), t^-_B(x)\}$ and

$1 - f^+_{A\cap B}(x) = \min\{1-f^+_A(x), 1-f^+_B(x)\}$,

$-1 - f^-_{A\cap B}(x) = \max\{-1-f^-_A(x), -1-f^-_B(x)\}$.

3. $\overline{A} = \{(x, [f^+_A(x), 1-t^+_A(x)], [-1-t^-_A(x), f^-_A(x)]) / x \in X\}$ for all $x \in X$.

Definition 3.2: Let A and B be two bipolar vague sets defined over a universe of discourse X. We say that $A \subseteq B$ if and only if $t_A^+(x) \le t_B^+(x),\ 1-f_A^+(x) \le 1-f_B^+(x)$ and $t_A^-(x) \ge t_B^-(x), -1-f_A^+(x) \ge -1-f_B^-(x)$ for all $x \in X$.

Definition 3.3: A bipolar vague topology BVT on a nonempty set X is a family BV_τ of bipolar vague set in X satisfying the following axioms

1. $0, 1 \in BV_\tau$
2. $G_1 \cap G_2 \in BV_\tau$, for any $G_1, G_2 \in BV_\tau$
3. $\cup G_i \in BV_\tau$, for any arbitrary family $\{G_i : G_i \in BV_\tau, i \in I\}$.

In this case the pair (X, BV_τ) is called a bipolar vague topological space and any BVS in BV_τ is known as bipolar vague open set in X. The complement $\overline{A}$ of a bipolar vague open set (BVOS) A in a bipolar vague topological space (X, BV_τ) is called a bipolar vague closed (BVCS) in X.

Example 3.4: Let $X = \{a,b\},\ A = \langle x, \left(\frac{a}{[0.4,0.6][-0.6,-0.5]}, \frac{b}{[0.6,0.8][-0.6,-0.3]} \right) \rangle$,

$B = \langle x, \left(\frac{a}{[0.2,0.4][-0.6,-0.4]}, \frac{b}{[0.4,0.7][-0.6,-0.2]} \right) \rangle$, $C = \langle x, \left(\frac{a}{[0.5,0.6][-0.7,-0.5]}, \frac{b}{[0.7,0.9][-0.7,-0.5]} \right) \rangle$.

Then the family $BV_\tau = \{0, 1, A, B, C\}$ of bipolar vague sets in X is a BVT on X.

Definition 3.5: Let (X, BV_τ) be a bipolar vague topological space and $A = \langle x, [t_A^+, 1-f_A^+][-1-f_A^-, t_A^-] \rangle$ be a BVS in X. Then the bipolar vague interior and bipolar vague closure of A are defined by,

$$Bvcl(A) = \cap \{K : K \text{ is a } BVCS \text{ in } X \text{ and } A \subseteq K\},$$
$$Bv\operatorname{int}(A) = \cup \{G : G \text{ is a } BVOS \text{ in } X \text{ and } G \subseteq A\}$$

Note that $bvcl(A)$ is a BVCS and $bv\operatorname{int}(A)$ is a BVOS in X. Further,

1. A is a BVCS in X iff $Bvcl(A) = A$,
2. A is a BVOS in X iff $Bv\operatorname{int}(A) = A$

Example 3.6: Let $X = \{a, b, c\}$,

$$A = \langle x, \left(\frac{a}{[0.4,0.6][-0.5,-0.4]}, \frac{b}{[0.3,0.7][-0.4,-0.3]}, \frac{c}{[0.5,0.6][-0.7,-0.5]} \right) \rangle$$
$$B = \langle x, \left(\frac{a}{[0.5,0.7][-0.6,-0.5]}, \frac{b}{[0.6,0.8][-0.5,-0.4]}, \frac{c}{[0.6,0.9][-0.8,-0.6]} \right) \rangle$$

Then the family $BV_\tau = \{0, 1, A, B\}$ of a bipolar vague sets in X is a BVT on X.

If $F = \langle x, \left(\frac{a}{[0.2,0.4][-0.6,-0.4]}, \frac{b}{[0.4,0.5][-0.4,-0.2]}, \frac{c}{[0.5,0.7][-0.7,-0.5]} \right) \rangle$ then

$Bv\operatorname{int}(F) = \cup \{G : G \text{ is a } BVOS \text{ in } X \text{ and } G \subseteq F\} = 0$, and
$Bvcl(F) = \cap \{K : K \text{ is a } BVCS \text{ in } X \text{ and } F \subseteq K\} = 1$.

Proposition 3.7: For any BVS A in (X, BV_τ) we have

1. $Bvcl(\overline{A}) = \overline{Bv\text{int}(A)}$.
2. $Bv\text{int}(\overline{A}) = \overline{Bvcl(A)}$.

Proof:

Let $A = \langle x,[t_A^+, 1-f_A^+],[-1-f_A^-, t_A^-]\rangle$ and suppose that $BVOS's$ contained in A are indexed by the family $\{< x,[t_{G_i}^+, 1-f_{G_i}^+],[-1-f_{G_i}^-, t_{G_i}^-]>: i \in J\}$ .Then, $Bv\text{int}(A) =< x,[\cup t_{G_i}^+, \cup 1-f_{G_i}^+],[\cap -1-f_{G_i}^-, \cap t_{G_i}^-]>$ and hence $\overline{Bv\text{int}(A)} =< x,[\cap f_{G_i}^+, \cap 1-t_{G_i}^+],[\cup -1-t_{G_i}^-, \cup f_{G_i}^-]>$ ……… (1)

Since $\overline{A} = \langle x,[f_A^+, 1-t_A^+],[-1-t_A^-, f_A^-]\rangle$ where $t_{G_i}^+ \le t_A^+$, $1-f_{G_i}^+ \le 1-f_A^+$ and $-1-f_{G_i}^- \ge -1-f_A^-$, $t_{G_i}^- \ge t_A^-$, for every $i \in J$ we obtain that $\{< x,[f_{G_i}^+, 1-t_{G_i}^+],[-1-t_{G_i}^-, f_{G_i}^-]>: i \in J\}$ is the family of $BVCS's$ containing $\overline{A}$, that is, $Bvcl(\overline{A}) =< x,[\cap f_{G_i}^+, \cap 1-t_{G_i}^+],[\cup -1-t_{G_i}^-, \cup f_{G_i}^-]>$ ………………. (2)

Hence from (1) and (2) we get $Bvcl(\overline{A}) = \overline{Bv\text{int}(A)}$.

(2) Follows from (1).

Proposition 3.8: Let (X, BV_τ) be a BVTS and A, B be are BVS's in X. Then the following properties hold:

1) $Bv\text{int}(A) \subseteq A$
2) $A \subseteq Bvcl(A)$
3) $A \subseteq B \Rightarrow Bv\text{int}(A) \subseteq Bv\text{int}(B)$
4) $A \subseteq B \Rightarrow Bvcl(A) \subseteq Bvcl(B)$
5) $Bv\text{int}(Bv\text{int}(A)) = Bv\text{int}(A)$
6) $Bvcl(Bvcl(A)) = Bvcl(A)$
7) $Bv\text{int}(A \cap B) = Bv\text{int}(A) \cap Bv\text{int}(B)$
8) $Bvcl(A \cup B) = Bvcl(A) \cup Bvcl(B)$
9) $Bv\text{int}(1) = 1$
10) $Bvcl(0) = 0$

Definition 3.9: Let (X, BV_τ) and (Y, BV_σ) be two bipolar vague topological spaces and $\psi : X \to Y$ be a function. Then ψ is said to be bipolar vague continuous iff the preimage of each bipolar vague open set in Y is a bipolar vague open set in X.

Proposition 3.10: Let A, $\{A_i : i \in J\}$ be a bipolar vague set in X, and B, $\{B_j : j \in K\}$ be a bipolar vague set in Y, and $\psi : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Leftrightarrow \psi(A_1) \subseteq \psi(A_2)$,
(b) $B_1 \subseteq B_2 \Leftrightarrow \psi^{-1}(B_1) \subseteq \psi^{-1}(B_2)$
(c) $\psi^{-1}(\cup B_i) = \cup \psi^{-1}(B_i)$ and $\psi^{-1}(\cap B_i) = \cap \psi^{-1}(B_i)$.

Proof: Obvious

Proposition 3.11: The following are equivalent to each other.

1. $\psi : X \to Y$ is bipolar vague continuous.
2. $\psi^{-1}(Bv\text{int}(B)) \subseteq Bv\text{int}(\psi^{-1}(B))$ for each BVOS B in Y.
3. $Bvcl(\psi^{-1}(B)) \subseteq \psi^{-1}(Bvcl(B))$ for each BVOS B in Y.

Proof: $(1)\Rightarrow(2)$ Given $\psi : X \to Y$ is bipolar vague continuous.

Then we have to show that $\psi^{-1}(Bv\text{int}(B)) \subseteq Bv\text{int}(\psi^{-1}(B))$ for each BVOS B in Y. Let $B = < y, [t_B^+, 1-f_B^+], [-1-f_B^-, t_B^-] >$ be a BVOS in Y and Let $Bv\text{int}(B) = \{< y, [\cup t_{H_i}^+, \cup 1-f_{H_i}^+][\cap -1-f_{H_i}^-, \cap t_{H_i}^-] >: i \in I\}$, where $t_{H_i}^+ \le t_B^+$, $1-f_{H_i}^+ \le 1-f_B^+$ and $-1-f_{H_i}^- \ge -1-f_B^-$, $t_{H_i}^- \ge t_B^-$, for every $i \in I$. By the definition of continuity $\psi^{-1}(Bv\text{int}(B))$ is a bipolar vague open set in BV_τ. Now,

$$\begin{aligned}\psi^{-1}(Bv\text{int}(B)) &= \{\psi^{-1}(< y, [\cup t_{H_i}^+, \cup 1-f_{H_i}^+][\cap -1-f_{H_i}^-, \cap t_{H_i}^-] >)\}\\ &= \{(< x, [\psi^{-1}(\cup t_{H_i}^+), \psi^{-1}(\cup 1-f_{H_i}^+)][\psi^{-1}(\cap -1-f_{H_i}^-), \psi^{-1}(\cap t_{H_i}^-)] >)\}\\ &= \{(< x, \cup[(\psi^{-1}(t_{H_i}^+)), \cup(\psi^{-1}(1-f_{H_i}^+))][\cap(\psi^{-1}(-1-f_{H_i}^-)), \cap(\psi^{-1}(t_{H_i}^-))] >)\}\\ &\subseteq Bv\text{int}(\psi^{-1}(B)).\end{aligned}$$

$(2)\Rightarrow(1)$ Given $\psi^{-1}(Bv\text{int}(B)) \subseteq Bv\text{int}(\psi^{-1}(B))$, for each BVOS B in Y. Let $B = < y, [t_B^+, 1-f_B^+][-1-f_B^-, t_B^-] >$ be a BVOS in Y. We know that B is bipolar vague open in Y if and only if $Bv\text{int}(B) = B$ and hence $\psi^{-1}(Bv\text{int}(B)) = \psi^{-1}(B)$. But according to our assumption $\psi^{-1}(Bv\text{int}(B)) \subseteq Bv\text{int}(\psi^{-1}(B))$, therefore we get $\psi^{-1}(B) \subseteq Bv\text{int}(\psi^{-1}(B))$. Hence $\psi^{-1}(B) = Bv\text{int}(\psi^{-1}(B)$, i.e., $\psi^{-1}(B)$ is a BVS in X and this proves that ψ is a bipolar vague continuous .

$(1)\Rightarrow(3)$ Given $\psi : X \to Y$ is bipolar vague continuous.

Let $B = < y, [t_B^+, 1-f_B^+][-1-f_B^-, t_B^-] >$ be a BVOS in Y.
Let $Bvcl(B) = \{< y, [\cap t_{K_i}^+, \cap 1-f_{K_i}^+], [\cup -1-f_{K_i}^-, \cup t_{K_i}^-] >: i \in I\}$,
where $t_{K_i}^+ \ge t_B^+$, $1-f_{K_i}^+ \ge 1-f_B^+$
and $-1-f_{K_i}^- \le -1-f_B^-$, $t_{K_i}^- \le t_B^-$, for each $i \in I$. Since ψ is a bipolar vague continuous iff the inverse image of each BVCS in Y is a BVCS in X, therefore $\psi^{-1}(Bvcl(B))$ is a BVCS in X Now,

$$\begin{aligned}\psi^{-1}(Bvcl(B)) &= \{\psi^{-1}(< y, [\cap t_{K_i}^+, \cap 1-f_{K_i}^+][\cup -1-f_{K_i}^-, \cup t_{K_i}^-] >)\}\\ &= \{(< x, [\psi^{-1}(\cap t_{K_i}^+), \psi^{-1}(\cap 1-f_{K_i}^+)][\psi^{-1}(\cup -1-f_{K_i}^-), \psi^{-1}(\cup t_{K_i}^-)] >)\}\\ &= \{(< x, \cap[(\psi^{-1}(t_{K_i}^+)), \cap(\psi^{-1}(1-f_{K_i}^+))][\cup(\psi^{-1}(-1-f_{K_i}^-)), \cup(\psi^{-1}(t_{K_i}^-))] >)\}\\ &\supseteq Bvcl(\psi^{-1}(B)).\end{aligned}$$

$(3)\Rightarrow(1)$ Given $Bvcl(\psi^{-1}(B)) \subseteq \psi^{-1}(Bvcl(B))$, for each BVOS B in Y.Let $B = < y, [t_B^+, 1-f_B^+][-1-f_B^-, t_B^-] >$ be a BVCS in Y. Since $Bvcl(B) = B$. But it is given that $Bvcl(\psi^{-1}(B)) \subseteq \psi^{-1}(Bvcl(B))$, hence $Bvcl(\psi^{-1}(B)) \subseteq \psi^{-1}(B)$ .Hence $\psi^{-1}(B) = Bvcl(\psi^{-1}(B)$, i.e., $\psi^{-1}(B)$ is a BVCS in X and this proves that ψ is a bipolar vague continuous .

4. GENERALIZED BIPOLAR VAGUE CLOSED SETS

Definition 4.1: Let (X, BV_τ) be a bipolar vague topological space. A bipolar vague set A in (X, BV_τ) is said to be a generalized bipolar vague closed set if $Bvcl(A) \subseteq G$ whenever $A \subseteq G$ and G is bipolar vague open. The complement of a generalized bipolar vague closed set is generalized bipolar vague open set.

Definition 4.2: Let (X, BV_τ) be a bipolar vague topological space and A be a bipolar vague set in X. Then the generalized bipolar vague closure $(GBvcl\ \textit{for short})$ and generalized bipolar vague interior $(GBv\text{int}\ \textit{for short})$ of A are defined by,

1) $GBvcl(A) = \cap\{G : G\ \textit{is a generalized bipolar vague closed set in}\ X\ \textit{and}\ A \subseteq G\}$,

2) $GB\,\text{int}(A) = \cup\{G : G \text{ is a generalized bipolar vague open set in } X \text{ and } A \supseteq G\}$

Remark 4.3: Every BVCS is generalized bipolar vague closed but not conversely.

Example 4.4: Let $X = \{a, b\}$ and $BV_\tau = \{0, 1, G\}$ is a BVT on X where

$G =< x, \left(\dfrac{a}{[0.4, 0.6][-0.4, -0.2]}, \dfrac{b}{[0.2, 0.4][-0.5, -0.3]} \right) >$ .Then bipolar vague set

$A =< x, \left(\dfrac{a}{[0.5, 0.6][-0.3, -0.2]}, \dfrac{b}{[0.5, 0.8][-0.5, -0.2]} \right) >$ is a generalized bipolar vague closed but not BVC in X .

Proposition 4.5: Let (X, BV_τ) be a bipolar vague topological space. If A is a generalized bipolar vague closed set and $A \subseteq B \subseteq Bvcl(A)$, then B is a generalized bipolar vague closed set.

Proof: Let G be a bipolar vague open set in (X, BV_τ), such that $B \subseteq G$. Since $A \subseteq B$, $A \subseteq G$. Now A is a generalized bipolar vague closed set and $Bvcl(A) \subseteq G$. But $Bvcl(B) \subseteq Bvcl(A)$. Since $Bvcl(B) \subseteq Bvcl(A) \subseteq G$, $Bvcl(B) \subseteq G$. Hence, B is a generalized bipolar closed set.

Proposition 4.6: If A is a bipolar vague open set and generalized bipolar vague closed set in (X, BV_τ), then A is a bipolar vague closed set in X .

Proof: Let A is a bipolar vague open set in X. Since $A \subseteq A$, by hypothesis $Bvcl(A) \subseteq A$. But from definition $A \subseteq Bvcl(A)$. Therefore $Bvcl(A) = A$. Hence A is bipolar vague closed set in X .

Proposition 4.7: Let (X, BV_τ) be a bipolar vague topological space. A bipolar vague set A is a generalized vague open set if and only if $B \subseteq Bv\text{int}(A)$, whenever B is a bipolar vague closed set and $B \subseteq A$.

Proof: Let A be a generalized bipolar vague open set and B be a bipolar vague closed set, such that $B \subseteq A$. Now, $B \subseteq A \Rightarrow \overline{A} \subseteq \overline{B}$ and $\overline{A}$ is a generalized bipolar vague closed set implies that $Bvcl(\overline{A}) \subseteq \overline{B}$. That is, $B = \overline{(\overline{B})} \subseteq \overline{Bvcl(\overline{A})}$. But $\overline{Bvcl(\overline{A})} = Bv\text{int}(A)$. Thus, $B \subseteq Bv\text{int}(A)$

Conversely, suppose that A is a bipolar vague set, such that $B \subseteq Bv\text{int}(A)$, whenever B is a bipolar vague closed set and $B \subseteq A$. Let $\overline{A} \subseteq B$ whenever B is a bipolar vague open set. Now, $\overline{A} \subseteq B \Rightarrow \overline{B} \subseteq A$. Hence by assumption, $\overline{B} \subseteq Bv\,\text{int}(A)$. That is, $\overline{Bv\text{int}(A)} \subseteq B$. But $\overline{Bv\text{int}(A)} = Bvcl(\overline{A})$. Hence, $Bvcl(\overline{A}) \subseteq B$. That is, $\overline{A}$ is a generalized bipolar vague closed set. Therefore, A is a generalized bipolar vague open set.

Proposition 4.8: If $Bv\text{int}(A) \subseteq B \subseteq A$ and if A is a generalized bipolar vague open set then B is also a generalized bipolar vague open set.

Proof: Now, $\overline{A} \subseteq \overline{B} \subseteq \overline{Bv\,\text{int}(A)} = Bvcl(\overline{A})$. As A is a generalized bipolar vague open, $\overline{A}$ is generalized bipolar vague closed set. By proposition 4.5, $\overline{B}$ is a generalized bipolar vague closed set. That is, B is a generalized bipolar vague open set.

Definition 4.9: Let (X, BV_τ) and (Y, BV_σ) be any two bipolar vague topological spaces

1. A map $\psi : (X, BV_\tau) \to (Y, BV_\sigma)$ is said to be generalized bipolar vague continuous if the inverse image of every bipolar vague open set in (Y, BV_σ) is a generalized vague open set in (X, BV_τ) .

2. A map $\psi : (X, BV_\tau) \to (Y, BV_\sigma)$ is said to be generalized bipolar vague irresolute if the inverse image of every generalized bipolar vague open in (Y, BV_σ) is a generalized bipolar vague open set in (X, BV_τ).

Proposition 4.10: Let (X, BV_τ) and (Y, BV_σ) be any two bipolar vague topological spaces. Let $\psi : (X, BV_\tau) \to (Y, BV_\sigma)$ be a generalized bipolar vague continuous function mapping. Then for every bipolar vague set A in X, $\psi(GBvcl(A)) \subseteq Bvcl(\psi(A))$.

Proof: Let A be a bipolar vague set in (X, BV_τ). Since $Bvcl(\psi(A))$ is a bipolar vague closed set and ψ is a generalized bipolar vague continuous mapping, $\psi^{-1}(Bvcl(\psi(A)))$ is a generalized bipolar vague closed set and $\psi^{-1}(Bvcl(\psi(A))) \supseteq A$. Now, $GBvcl(A) \subseteq \psi^{-1}(Bvcl(\psi(A)))$. Therefore, $\psi(GBvcl(A)) \subseteq Bvcl(\psi(A))$.

Proposition 4.11: Let (X, BV_τ) and (Y, BV_σ) be any two bipolar vague topological spaces. Let $\psi : (X, BV_\tau) \to (Y, BV_\sigma)$ be a generalized bipolar vague continuous mapping. Then for every bipolar vague set A in Y, $GBvcl(\psi^{-1}(A)) \subseteq \psi^{-1}(Bvcl(A))$.

Proof: Let A be a bipolar vague set in (Y, BV_σ). Let $B = \psi^{-1}(A)$. Then, $\psi(B) = \psi(\psi^{-1}(A)) \subseteq A$, By proposition 4.10, $\psi(GBvcl(\psi^{-1}(A))) \subseteq Bvcl(\psi(\psi^{-1}(A)))$. Thus, $GBvcl(\psi^{-1}(A)) \subseteq \psi^{-1}(Bvcl(A))$.

Proposition 4.12: Let (X, BV_τ) and (Y, BV_σ) be any two bipolar vague topological spaces. If $\psi : (X, BV_\tau) \to (Y, BV_\sigma)$ is a bipolar vague continuous mapping, then it is a generalized bipolar vague continuous mapping.

Proof: Let A be a bipolar vague open set in (Y, BV_σ). Since ψ is a bipolar vague continuous mapping, $\psi^{-1}(A)$ is a bipolar vague open set in (X, BV_τ). Every bipolar vague open set is a generalized bipolar vague open set. Now, $\psi^{-1}(A)$ is a generalized bipolar vague open set in (X, BV_τ). Hence, ψ is a generalized bipolar vague continuous mapping.

The converse of the above need not be true as shown in example

Example 4.13: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $A = < x, \frac{a}{[0.4, 0.7][-0.6, -0.3]}, \frac{b}{[0.2, 0.5][-0.7, -0.4]} >$, $B = < y, \frac{a}{[0.5, 0.7][-0.6, -0.2]}, \frac{b}{[0.4, 0.7][-0.5, -0.3]} >$.

Then $BV_\tau = \{0, 1, A\}$ and $BV_\sigma = \{0, 1, B\}$ are BVT on X and Y respectively. Define a mapping $\psi : (X, BV_\tau) \to (Y, BV_\sigma)$ by $\psi(a) = u$ an $\psi(b) = v$. Then ψ is generalized bipolar vague continuous mapping but not bipolar vague continuous mapping.

Proposition 4.14: Let (X, BV_τ) and (Y, BV_σ) be any two bipolar vague topological spaces. If $\psi : (X, BV_\tau) \to (Y, BV_\sigma)$ is a generalized bipolar vague irresolute mapping, then it is a generalized bipolar vague continuous mapping.

Proof: Let A be a bipolar vague open set in (Y, BV_σ). Since every bipolar vague open is generalized bipolar vague open set in (Y, BV_σ), but ψ is a generalized bipolar vague irresolute mapping, $\psi^{-1}(A)$ is a generalized bipolar vague open set in (X, BV_τ). Hence, ψ is a generalized bipolar vague continuous mapping.

Proposition 4.15: Let (X, BV_τ), (Y, BV_σ) and (Z, BV_ρ) be any three bipolar vague topological spaces. Let $\psi : (X, BV_\tau) \to (Y, BV_\sigma)$ be a generalized bipolar vague irresolute mapping and $\psi_1 : (Y, BV_\sigma) \to (Z, BV_\rho)$ be a generalized bipolar vague continuous mapping. Then $\psi_1 \circ \psi$ is a generalized bipolar vague continuous mapping.

Proof: Let A be a bipolar vague open set in (Z, BV_{ρ}). Since ψ_1 is a generalized bipolar vague continuous mapping, $\psi_1^{-1}(A)$ is a generalized bipolar vague open set in (Y, BV_{σ}). Since ψ is a generalized bipolar vague irresolute mapping, $\psi^{-1}(\psi_1^{-1}(A))$ is a generalized bipolar open set in (X, BV_{τ}). Thus, $\psi_1 \circ \psi$ is a generalized bipolar vague continuous mapping.

REFERENCES

[1]Arockiarani, I. and Cicily Flora. S. 2016. Positive Implicative Bipolar Vague Ideals In BCK- algebras, International Research Journal of pure Algebra, 6(8), 1-7.
[2]Balasubramanian, G., Sundaram, P. 1997. On some generalization of fuzzy continuous functions, Fuzzy Sets and Systems, 86, 93–100.
[3]Chang. C.L. 1986. Fuzzy topological space, J. Math. Anal. Appl, 24, 182-190.
[4]Cicily Flora. S and Arockiarani, I, A View On Bipolar Vague ring in baire space(To Appear).
[5]Coker, D. 1997. An Introduction to Intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems, 88, 81-89.
[6]Coker, D., Gurcay, H. and Hyder, A. 1997. On Fuzzy Continuity in Intuitionistic Fuzzy Topological Spaces, The Journal of Fuzzy Mathematics, 5(2), 365-378.
[7]Gau. W. L, Buehrer. D.J. 1993. Vague sets, IEEE Transcations on Systems, Man and Cybernatics, 23(2), 610-614.
[8]Levine. N, Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19, 1970, 89-96.
[9]Lee. K.M 2004. Comparison of interval-valued fizzy sets, intuitionistic fuzzy sets and bipolar-valued fuzzy sets, J. Fuzzy Logic Intelligent, 14(2), 125-129.
[10]Lee. K.M. 2000. Bipolar-valued fuzzy sets and their operations, Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand, 307-312.
[11]Zadeh. L.A. 1965. Fuzzy sets, Information and Control, 8, 338-353.

****